

Evaluating the Stolt-stretch parameter^a

^aPublished in Journal of Seismic Exploration, v. 9, 319-335, (2001)

Sergey Fomel¹ and Louis Vaillant^{2,1}

ABSTRACT

The Stolt migration extension to a variable velocity case describes the velocity heterogeneity with a constant parameter, which is related to the stretch transformation of the time axis. We exploit a connection between modified dispersion relations and nonhyperbolic traveltime approximations to derive an explicit expression for the stretch parameter. This analytical expression allows one to achieve the highest possible accuracy within the Stolt stretch approximation. Using a real data example, we demonstrate an application of the explicit Stolt stretch formula for an optimal partitioning of the migration velocity in the method of cascaded migrations.

INTRODUCTION

Although Stolt migration is regarded as the fastest of all the known seismic migration algorithms, it has a limited applicability because of the intrinsic constant velocity assumption. The time-stretching trick proposed in Stolt's classic paper (Stolt, 1978) provides an approximate extension of the method to a variable velocity case. Implicitly, Stolt stretch transforms reflection traveltime curves to fit an approximate constant velocity pattern (Levin, 1983, 1985; Claerbout, 1985). In other words, the wave equation with variable velocity is transformed by a particular stretch of the time axis to an approximate differential equation with constant coefficients. The two constant coefficients are an arbitrarily chosen frame velocity and a special non-dimensional parameter (W in Stolt's original notation). In the constant velocity case W is equal to 1, and the transformed equation coincides with the exact constant velocity wave equation. In variable velocity media, W is generally assumed to lie between 0 and 1. As shown by Larner and Beasley (1987), the cascaded f - k migration approach can move the value of W for each migration in a cascade closer to 1, thus increasing the accuracy of the Stolt stretch approximation.

¹*Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Mail Stop 50A-1148, Berkeley, California 94720, USA, formerly Stanford Exploration Project, Department of Geophysics, Stanford University, Stanford, California 94305, USA*

²*UNILOG - DO SCL, 37 Rue du Rocher, Paris 75008, France, formerly Stanford Exploration Project, Department of Geophysics, Stanford University, Stanford, California 94305, USA*

¹**e-mail:** sergey@sep.stanford.edu

The W factor is defined by Stolt (1978) as an approximate average of a complicated function, which depends on both time and space coordinates and cannot be computed directly. Therefore, in practice, the estimation of W is always replaced by a heuristic guess. That is why Levin (1983) jokingly called the W parameter “infamous”, and Larner and Beasley (1987) called it “esoteric.”

In this paper, we use an analytic technique to evaluate the Stolt stretch parameter explicitly. The main idea is to constrain this parameter by fitting the Stolt-stretch traveltimes function to the exact one. It turns out that in the isotropic case, the W parameter is connected to the “parameter of heterogeneity” (Malovichko, 1978; Sword, 1987; Castle, 1988; de Bazelaire, 1988). The definition of heterogeneity is modified for the case of an anisotropic (transversally isotropic) media.

We demonstrate an application of the Stolt stretch analytical expression on a real data example from the North Sea. The velocity profile is optimally partitioned for the method of cascaded migration, which allows us to image steeply dipping reflectors at the accuracy comparable to that of the phase-shift method but at a much smaller cost.

Although Stolt migration is not currently at the forefront of geophysical research, it is still widely used in practice (Yilmaz, 2001; Yilmaz et al., 2001) and keeps recurring in different contexts. Popovici et al. (1996) propose a new interpolation scheme for improving the practical accuracy of the method. Sava (2000) uses a variation of Stolt migration - Stolt residual migration (Stolt, 1996) - in the context of wave-equation migration velocity analysis.

The growth in computer speed does not automatically make fast algorithms obsolete, because the amount of processed data tends to grow at the same rate or even faster. The researchers working in the field of seismic imaging are often interested in the following questions: What is the fastest possible migration algorithm? How accurate can it get? Stolt migration answers the first question. The answer to the second question is developed in this paper.

STOLT STRETCH THEORY REVIEW

In order to simplify the references, we start with definitions of the Stolt migration method. The reader familiar with the Stolt stretch theory can skip this section and go on to new theoretical results in the next section.

The basic migration theory reduces post-stack migration to a two-stage process. The first stage is a downward continuation of the wavefield in depth z based on the wave equation

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} = \frac{1}{v^2(x, z)} \frac{\partial^2 P}{\partial t^2}. \quad (1)$$

The second stage is the imaging condition $t = 0$ (here the velocity v is twice as small as the actual wave velocity). Stolt time migration performs both stages in one step,

applying the frequency-domain operator

$$\tilde{P}_0(k_x, \omega_0) = \tilde{P}_v(k_x, \omega_v(k, \omega_0)) \left| \frac{d\omega_v(k, \omega_0)}{d\omega_0} \right|, \quad (2)$$

where

$$\begin{aligned} \tilde{P}_v(k_x, \omega_v) &= \iint P_v(x, t_v) \exp(i\omega_v t_v - ik_x x) dt_v dx, \\ \tilde{P}_0(k_x, \omega_0) &= \iint P_0(x, t_0) \exp(i\omega_0 t_0 - ik_x x) dt_0 dx, \end{aligned}$$

$P_0(x, t_0)$ stands for the initial zero-offset (stacked) seismic section defined on the surface $z = 0$, $P_v(x, t_v)$ is the time-migrated section, and t_v is the vertical traveltime

$$t_v = \int_0^z \frac{dz'}{v(x, z')}. \quad (3)$$

The function $\omega_v(k, \omega_0)$ in (2) corresponds to the dispersion relation of the wave equation (1) and in the constant velocity case has the explicit expression

$$\omega_v(k, \omega_0) = \text{sign}(\omega_0) \sqrt{\omega_0^2 - v^2 k^2}. \quad (4)$$

The choice of the sign in equation (4) is essential for distinguishing between upgoing and downgoing waves. The upgoing part of the wavefield is the one used in migration.

The case of a varying velocity complicates the frequency-domain algorithm and therefore requires special consideration. Stolt (1978) suggested the following change of the time variable (referred to in the literature as *Stolt stretch*):

$$s(t) = \left(\frac{2}{v_0^2} \int_0^t \eta d\tau \right)^{1/2}, \quad (5)$$

where v_0 is an arbitrarily chosen constant velocity, and η is a function defined by the parametric expressions

$$\eta(\zeta) = \int_0^\zeta v(x, z) dz, \quad \tau(\zeta) = \int_0^\zeta \frac{dz}{v(x, z)}. \quad (6)$$

Applying equation (5), we can connect seismic time migration to the transformed wave equation

$$\frac{\partial^2 P}{\partial x^2} + W \frac{\partial^2 P}{\partial \hat{z}^2} + 2 \frac{(1-W)}{v_0} \frac{\partial^2 P}{\partial \hat{z} \partial \hat{t}} = \frac{(2-W)}{v_0^2} \frac{\partial^2 P}{\partial \hat{t}^2}. \quad (7)$$

The variables \hat{z} and \hat{t} correspond to the transformed depth and time coordinates, which possess the following property: if $\hat{z} = 0$, $\hat{t} = s(t_0)$, and if $\hat{t} = 0$, $\hat{z} = v_0 s(t_v)$. W is a varying coefficient defined as

$$W = a^2 + 2b(1 - a^2), \quad (8)$$

where

$$b = \frac{\eta(z)}{\eta(\zeta)}, \quad a = \frac{s(\tau) v_0 v(x, z)}{\eta(\zeta)}, \quad \tau = \int_0^\zeta \frac{dz}{v(x, z)} = t + \int_0^z \frac{dz'}{v(x, z')}.$$

Since the W parameter varies slowly with x and \hat{z} , Stolt suggested to replace it with its average value. Thus equation (7) is then approximated by an equation with constant coefficients, which has the dispersion relation

$$\hat{\omega}_v(k, \hat{\omega}_0) = \left(1 - \frac{1}{W}\right) \hat{\omega}_0 + \frac{\text{sign}(\hat{\omega}_0)}{W} \sqrt{\hat{\omega}_0^2 - W v_0^2 k^2}. \quad (9)$$

As outlined above, Stolt's approximate method for migration in heterogeneous media consists of the following steps:

1. stretching the time variable according to equation (5),
2. interpolating the stretched time to a regular grid,
3. double Fourier transform,
4. f - k time migration by the operator (2) with the dispersion relation (9),
5. inverse Fourier transform,
6. inverse stretching (that is, shrinking) of the vertical time variable on the migrated section.

The value of W must be chosen prior to migration. According to Stolt's original definition (8), the depth variable z gradually changes in the migration process from zero to ζ , causing the coefficient b in (8) to change monotonically from 0 to 1. If the velocity v monotonically increases with depth, then $\eta''(z) = \frac{\partial v}{\partial z} \geq 0$, and the average value of b is

$$\bar{b} = \frac{1}{\zeta \eta(\zeta)} \int_0^\zeta \eta(z) dz \leq \frac{1}{\zeta \eta(\zeta)} \int_0^\zeta \eta(\zeta) \frac{z}{\zeta} dz = \frac{1}{2}. \quad (10)$$

As follows from equations (8) and (10), in the case of monotonically increasing velocity, the average value of W has to be less than 1 (W equals 1 in a constant-velocity case). Analogously, in the case of a monotonically decreasing velocity, W is always greater than 1. In practice, W is included in migration routines as a user-defined parameter, and its value is usually chosen to be somewhere in the range of 1/2 to 1. The next section describes a straightforward way to determine the most appropriate value of W for a given velocity distribution.

A useful tool for that purpose is Levin's equation for the travelttime curve. Levin (1985) applied the stationary phase technique to the dispersion relation (9) to obtain

an explicit equation for the summation curve of the integral migration operator analogous to the Stolt stretch migration. The equation evaluates the summation path in the stretched coordinate system, as follows:

$$s(t_0) = \left(1 - \frac{1}{W}\right) s(t_v) + \frac{1}{W} \sqrt{s^2(t_v) + W \frac{(x - x_0)^2}{v_0^2}}, \quad (11)$$

where x_0 is the midpoint location on a zero-offset seismic section, and x is the space coordinate on the migrated section. Equation (11) shows that, with the stretch of the time coordinate, the summation curve has the shape of a hyperbola with the apex at $\{x, s(t_v)\}$ and the center (the intersection of the asymptotes) at $\{x, (1 - \frac{1}{W}) s(t_v)\}$. In the case of homogeneous media, $W = 1$, $s(t) \equiv t$, and equation (11) reduces to the known expression for a hyperbolic diffraction traveltime curve. It is interesting to note that inverting equation (11) for $s(t_v)$ determines the impulse response of the migration operator:

$$\hat{z} - \hat{z}_0 = \left(\frac{1}{Q} - 1\right) R \pm \frac{1}{Q} \sqrt{R^2 - Q(x - x_0)^2}, \quad (12)$$

where $R = v_0 \hat{t}$, and $Q = 2 - W$. Equation (12) can be interpreted as the wavefront from a point source in the $\{x, \hat{z}, \hat{t}\}$ domain of equation (7). Wavefronts from a point source in the stretched coordinates for $W < 2$ have an elliptic shape, with the center of the ellipse at $\{x, \hat{z}_0 + (\frac{1}{Q} - 1) R\}$ and the semi-axes $a_x = \frac{R}{\sqrt{Q}}$ and $a_z = \frac{r}{Q}$. The ellipses stretch differently for $W < 1$ and $W > 1$, as shown in Figure 1. In the upper part that corresponds to the upgoing waves, the ellipses look nearly spherical, since the radius of the front curvature at the top apex equals the distance from the source.

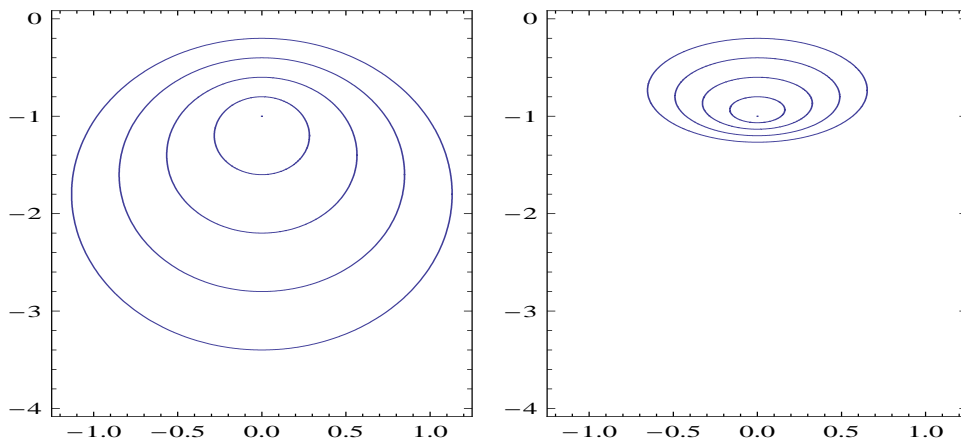


Figure 1: Wavefronts from a point source in the stretched coordinate system. Left: velocity decreases with depth ($W=1.5$). Right: velocity increases with depth ($W=0.5$).

EVALUATING THE W PARAMETER

A remarkable connection between the Stolt stretch equation and different three-parameter traveltimes approximations leads to a constructive estimate of the W parameter. The first useful observation is a formal similarity between equation (11) and Malovichko's approximation for the reflection traveltimes curve in vertically inhomogeneous media (Malovichko, 1978; Sword, 1987; Castle, 1988; de Bazelaire, 1988) defined by

$$t_0 = \left(1 - \frac{1}{S(t_v)}\right) t_v + \frac{1}{S(t_v)} \sqrt{t_v^2 + S(t_v) \frac{(x - x_0)^2}{v_{rms}^2(t_v)}}. \quad (13)$$

In equation (13), v_{rms} is the effective (root mean square) velocity along the vertical ray

$$v_{rms}^2(t_v) = \frac{\eta(z)}{t_v} = \frac{1}{t_v} \int_0^{t_v} v^2(t) dt, \quad (14)$$

and S is the *parameter of heterogeneity*, defined by the equation:

$$S(t_v) = \frac{1}{v_{rms}^4 t_v} \int_0^{t_v} v^4(t) dt. \quad (15)$$

In terms of the S parameter, the variance of the squared velocity distribution along the vertical ray is

$$\sigma^2 = \frac{1}{t_v} \int_0^{t_v} v^4(t) dt - v_{rms}^4 = v_{rms}^4 (S - 1). \quad (16)$$

As follows from equality (16), $S \geq 1$ for any type of velocity distribution (S equals 1 in a constant velocity case). For most of the distributions occurring in practice, S ranges between 1 and 2.

Since reflection from a horizontal reflector in vertically-heterogeneous media is kinematically equivalent to diffraction from a point, we can regard equation (13), which is known as the most accurate three-parameter approximation of the NMO curve, as an approximation of the summation path for the post-stack Kirchhoff migration operator. In this case, it has the same meaning as equation (11). An important difference between the two equations is the fact that equation (13) is written in the initial coordinate system and includes coefficients varying with depth, while equation (11) applies the transformed coordinate system and constant coefficients. Using this fact, we compare the accuracy of the approximations and derive the following explicit expression, which relates Stolt's W factor to Malovichko's parameter of heterogeneity:

$$W = 1 - \frac{v_0^2 s^2(t_v)}{v_{rms}^2(t_v) t_v^2} \left(\frac{v^2(t_v)}{v_{rms}^2(t_v)} - S(t_v) \right). \quad (17)$$

The details of the derivation are given in the appendix. Expression (17) is derived so as to provide the best possible value of W for a given depth (or vertical time t_v).

To get a constant value for a range of depths, one should take an average of the right-hand side of (17) in that range. The error associated with Stolt stretch can be approximately estimated from (A-1) as the difference between the fourth-order terms:

$$\delta = \frac{l^4}{8} \frac{W(t_v) - W}{t_v s^2(t_v) v_{rms}^2(t_v) v_0^2}, \quad (18)$$

where $W(t_v)$ is the right-hand side of (17), and W is the constant value of W chosen for Stolt migration.

Analytic Example

A simple analytic example is the case of a constant velocity gradient. In this case the velocity distribution can be described by the linear function $v(z) = v(0)(1 + \alpha z)$. The Stolt stretch transform for this case can be derived directly from equation (5) and takes the form

$$s(t) = \left(\frac{e^{2\alpha v(0)t} - 1 - 2\alpha v(0)t}{2\alpha^2 v_0^2} \right)^{1/2}. \quad (19)$$

Let κ be the logarithm of the velocity change $v(z)/v(0)$. Then an explicit expression for W factor is found according (17) as

$$W = \frac{2\kappa}{e^{2\kappa} - 1} = \frac{v^2(0)}{v_{rms}^2(z)}. \quad (20)$$

In the case of a small κ , which corresponds to a small depth or a small velocity gradient, $W \approx 1 - \kappa$. In the case of a large κ , W monotonically approaches zero. Equation (20) can be a useful rule of thumb for a rough estimation of W .

Stolt stretch for anisotropic media

As follows from the analysis of the reflection moveout in a vertically heterogeneous transversely isotropic medium (Fomel and Grechka, 1996), expression (17) for the Stolt stretch parameter will remain valid in this case if the values of v_{rms} and S are computed according to equations

$$v_{rms}^2(t_v) = \frac{1}{t_v} \int_0^{t_v} v^2(t) (1 + 2\delta(t)) dt, \quad (21)$$

$$S(t_v) = \frac{1}{v_{rms}^4 t_v} \int_0^{t_v} v^4(t) (1 + 2\delta(t))^4 (1 + 8\eta(t)) dt, \quad (22)$$

where δ and η are the conventional anisotropic parameters (Thomsen, 1986; Alkhalifah and Tsvankin, 1995), which may vary with depth.

As we demonstrate in the next section, the method of cascaded migrations (Larner and Beasley, 1987) can improve the performance of Stolt migration in the case of variable velocity (Beasley et al., 1988). However, this method affects only the isotropic part of the model and cannot change the contribution of the anisotropic parameters. Therefore, in the anisotropic case, it is important to incorporate anisotropic parameters into the Stolt stretch correction.

APPLICATION

Following the study by Larner et al. (1989), we selected a dataset that includes steep dips in order to test the accuracy of our algorithms. The dataset is courtesy of Elf Aquitaine. It was recorded in the North Sea over a salt-dome structure. Figure 2 shows the data after NMO-stack and after post-stack Stolt migration, using a constant velocity of 2000 m/s. The Stolt method creates visible undermigrated events on both sides of the salt body. Using a higher velocity to focus them better would have created overmigration artifacts at shallow reflectors. Stolt-stretch migrated section using $W = 0.5$ is shown in in Figure 2c. It should be compared with an improved result shown in Figure 3a.

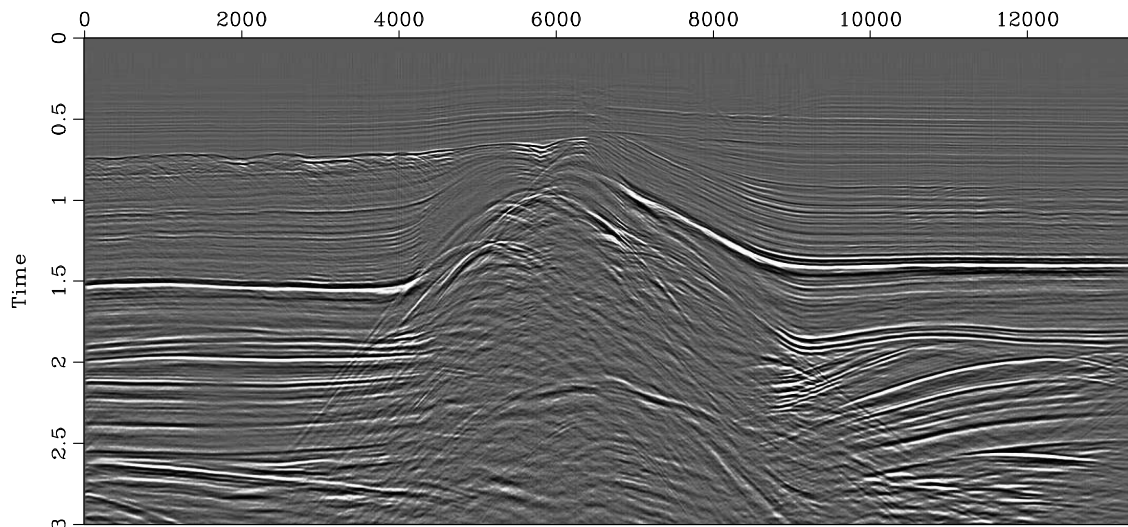
Using the Stolt-stretch method with the optimal choice for W estimated from equation (17) yields a better focusing of events at all depths (Figure 3a), compared to other values of W (Figures 2b and 2c, respectively for W equals 1.0 and 0.5). The $v(z)$ model used for migration is shown in Figure 4a and was obtained by averaging laterally the reference velocity model.

The reference method of migration for our study is the phase-shift method (Gazdag, 1978). It is known to be perfectly accurate for all dips up to 90° in a $v(z)$ velocity field. A comparison between the phase-shift migration result (Figure 3b) and the section migrated with the Stolt-stretch approach shows almost no difference for flat events. However, a more detailed analysis reveals significant errors for steep events inside and around the salt body. The approximation made by stretching the time axis breaks for recovering steep events.

A way to overcome the difficulties encountered by Stolt's migration is to divide the whole process into a cascade, as suggested by Beasley et al. (1988). The theory of cascaded migration proves that f - k migration algorithms with a $v(t)$ velocity model like Stolt-stretch can be performed sequentially as a cascade of n migrations with smaller interval velocities $v_i(t)$, $i = 1, \dots, n$, such then

$$v^2(t) = \sum_{i=1}^n v_i^2(t). \quad (23)$$

At a given vertical travelttime t , all the successive velocity models have to be constant, except the last one (Larner and Beasley, 1987). Typically, the first stage is done with a constant velocity model and can be computed using Stolt's method, which is then



(a) DMO-stack data

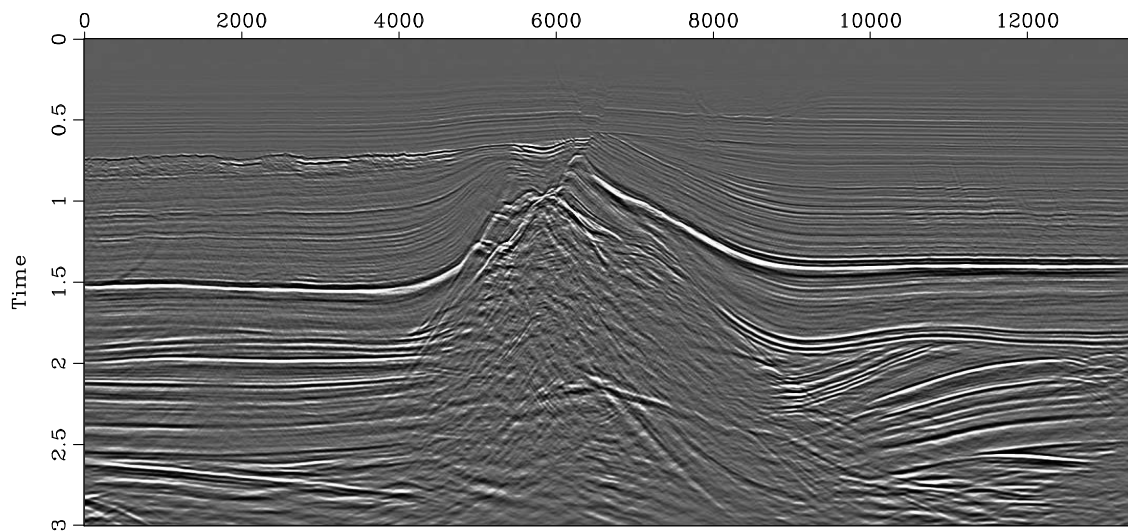
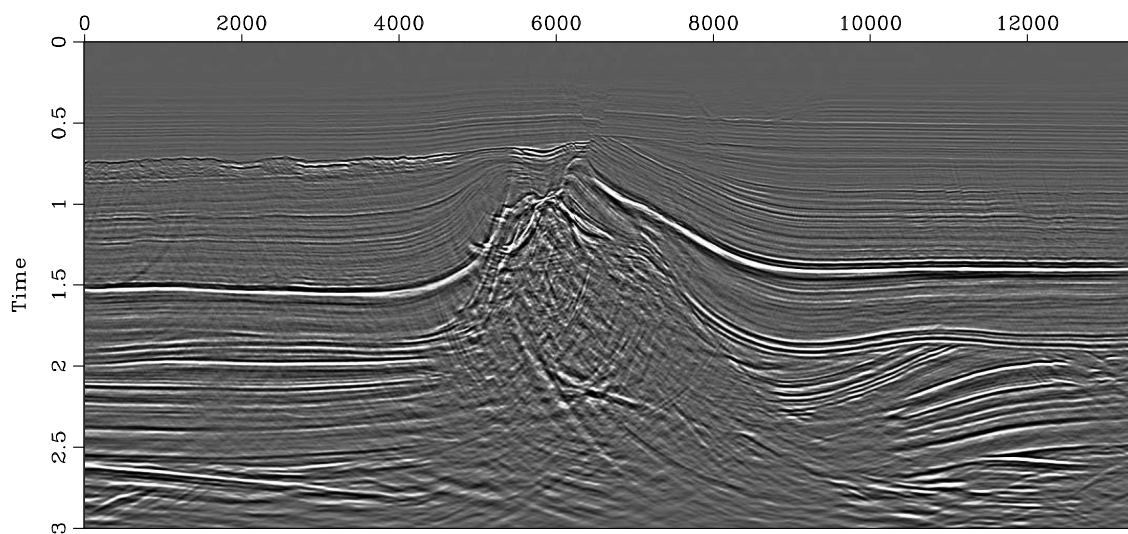
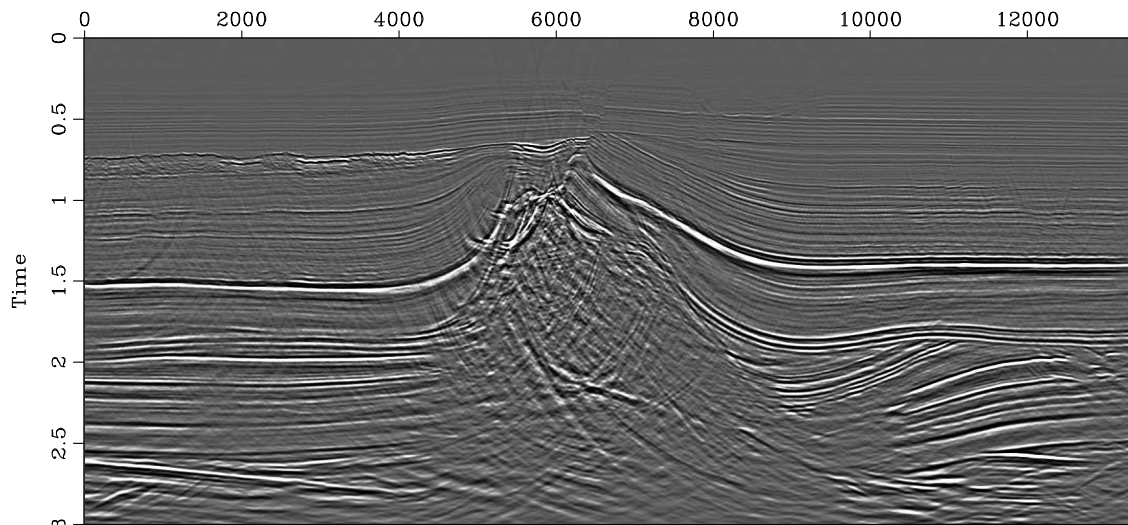
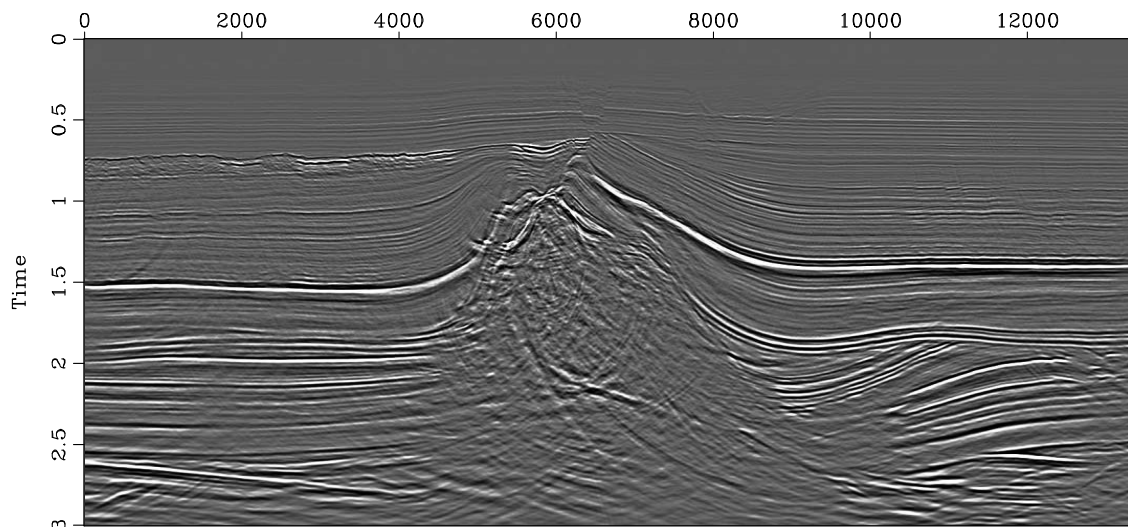
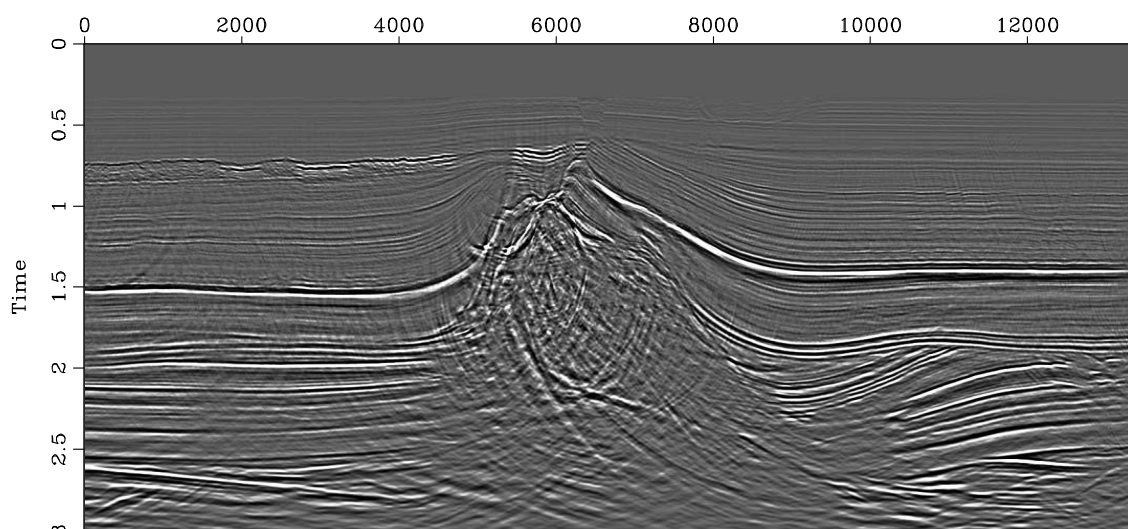
(b) Stolt migration ($v_0 = 2000$ m/s)(c) Stolt-stretch migration ($W=0.5$)

Figure 2: (a) Section of the North Sea data, after NMO-stack. (b) Section migrated using Stolt's method with $v_0=2000$ m/s. (c) Section migrated using Stolt-stretch with an arbitrary value $W = 0.5$ for the parameter of heterogeneity.

(a) Stolt-stretch migration (optimal W)

(b) Phase-shift migration



(c) Cascaded Stolt-Stretch (5 velocities)

Figure 3: (a) Section migrated with the Stolt-stretch method using the optimal value (≈ 0.67) for the parameter W . (b) Section migrated with the phase-shift method. (c) Section migrated using the cascaded Stolt-stretch approach (5 velocities).

accurate for all dips. Figure 4 illustrates such a cascade of velocity models in our particular case, with 3 and 5 stages.

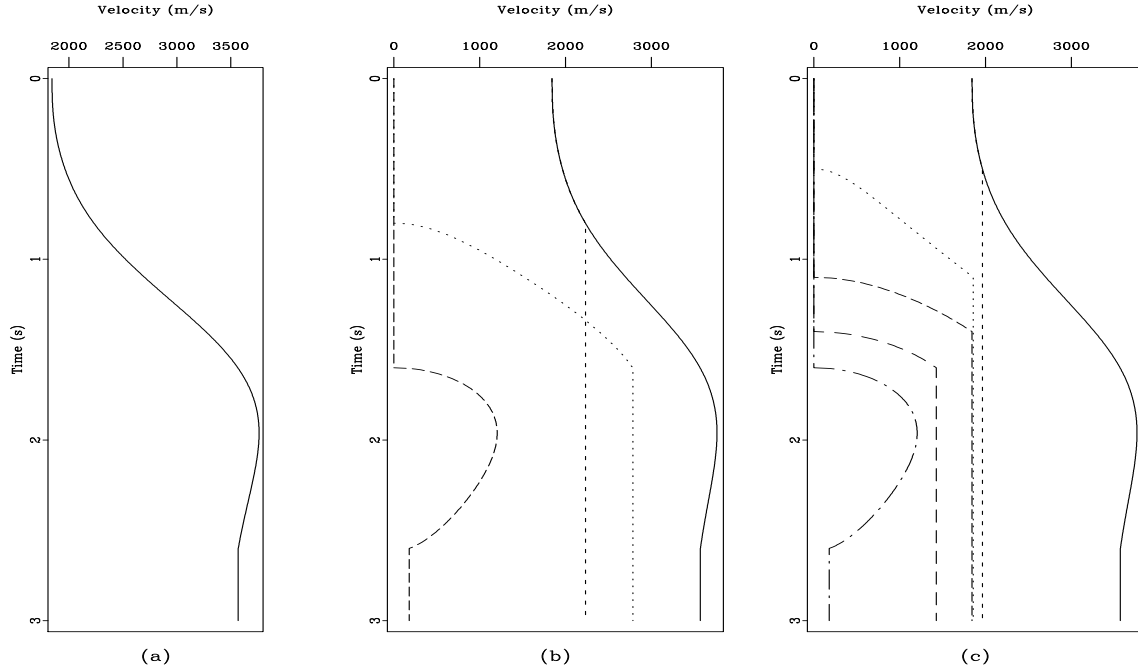


Figure 4: (a) Interval velocity model $v(t)$ estimated from the 2-D reference model. (b) Decomposition in a cascade of 3 models, such as $v^2 = v_1^2 + v_2^2 + v_3^2$. (c) Decomposition in a cascade of 5 models, such as $v^2 = v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2$

As a consequence of this decomposition, each intermediate velocity model shows not only a smaller velocity but also less vertical heterogeneity. In other words, the Stolt-stretch parameter W estimated for each stage tends to be closer to 1.0, thus reducing the migration errors due to the approximation. Figure 3c shows the migration result using a 5-stage cascaded scheme. All the successive values of W were greater than 0.8. There are almost no differences with the phase-shift result (Figure 3b).

An accuracy of the cascaded stolt-stretch migration is additionally verified by comparing its impulse response with that of the phase-shift migration (Figure 5). The impulses are generated using the same velocity model as shown in Figure 4a. Figure 6 provides a more detailed comparison. We can see a kinematic difference in the impulse response of Stolt-stretch compared to phase-shift. While Gazdag's phase-shift honors ray bending in any $v(z)$ model, Stolt-stretch is only designed to make the fitting curve look like an hyperbola close to the apex (Levin, 1983), and therefore induces residual migration errors. As seen in Figure 3a, Stolt-stretch result displays residual hyperbolic migration artifacts that are due to this fundamental kinematic difference. Cascading Stolt-stretch makes the impulse response of the migration converge towards the one of phase-shift.

Figure 7 shows a close-up of the salt body region for all migration algorithms. The methods have a different accuracy with respect to steep dips. We notice a gradual

improvement of the result from Stolt-stretch to phase-shift as we increase the number of velocities in the cascaded Stolt-stretch scheme. In theory, the migration errors in the cascaded approach can be made as small as desired by increasing the number of stages. At the limit, it corresponds to the velocity continuation concept (Fomel, 1994, 1997).

In our case, six stages were enough to obtain a result comparable to phase-shift. In their comparative study on time migration algorithms, Larner et al. (1989) have shown that four-stage cascaded f - k migration is accurate for dips up to 85° , which is almost comparable to phase-shift, accurate for all dips. It is worth noting the computational cost difference between the two: on our example, phase-shift migration is about 80 times more expensive than Stolt-stretch.

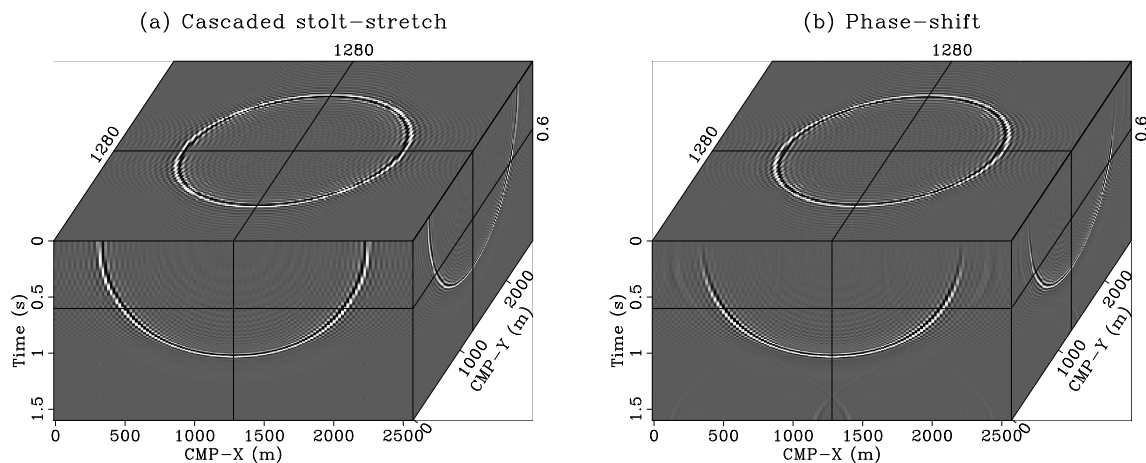


Figure 5: 3-D impulses responses of the cascaded Stolt-stretch (a) and phase-shift (b) operators.

CONCLUSIONS

An explicit expression for the Stolt-stretch parameter, derived in this paper, allows us to achieve optimal accuracy when applying Stolt migration in vertically heterogeneous media.

Combining an optimal analytical choice for the Stolt-stretch parameter with the cascaded f - k migration approach, we manage to obtain time migration results comparable to Gazdag's phase-shift migration. The Stolt method is considerably more computer-efficient and remains accurate for steeply dipping events.

ACKNOWLEDGMENTS

The authors would like to thank the sponsors of the Stanford Exploration Project for financial support and Elf Aquitaine for providing the data used in this work.

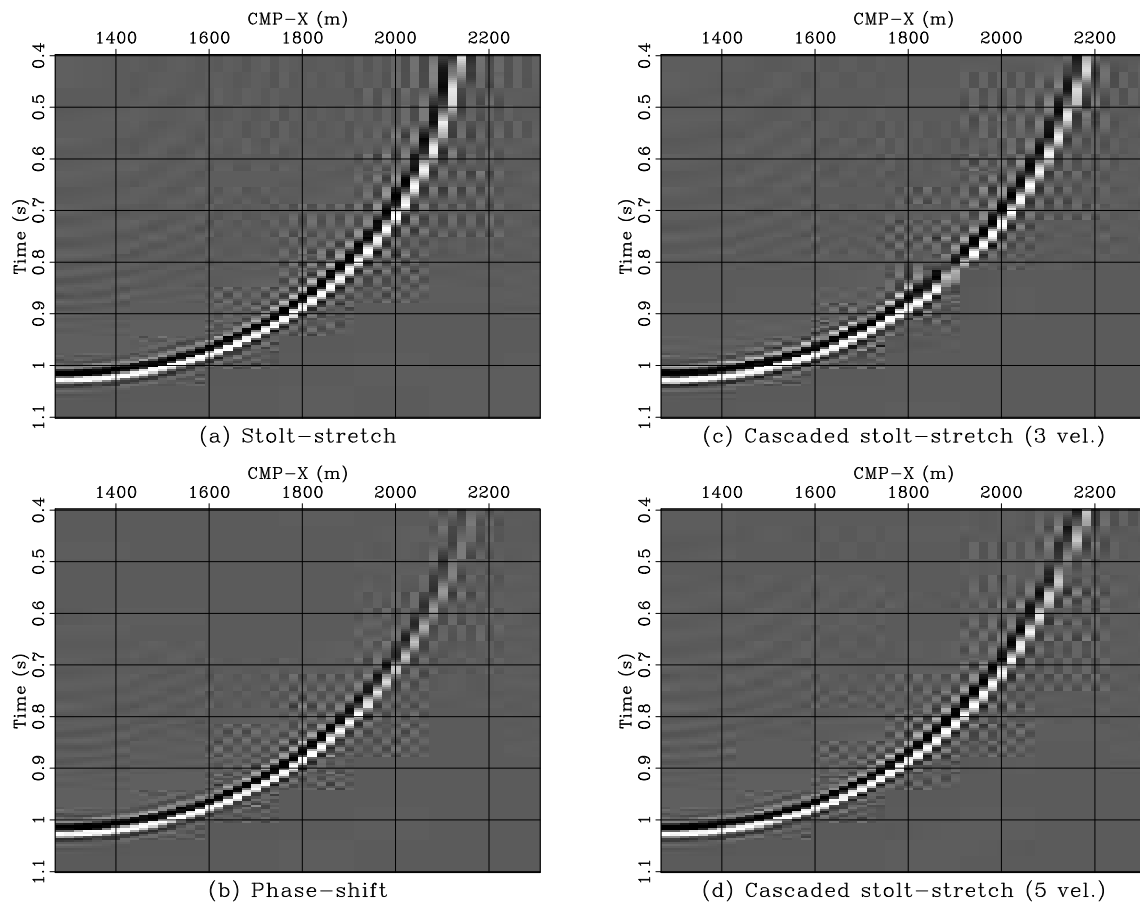


Figure 6: Impulse responses of the different operators. (a) Stolt-stretch. (b) Phase-shift. (c) and (d) Cascaded Stolt-stretch, with 3 and 5 velocities, respectively.

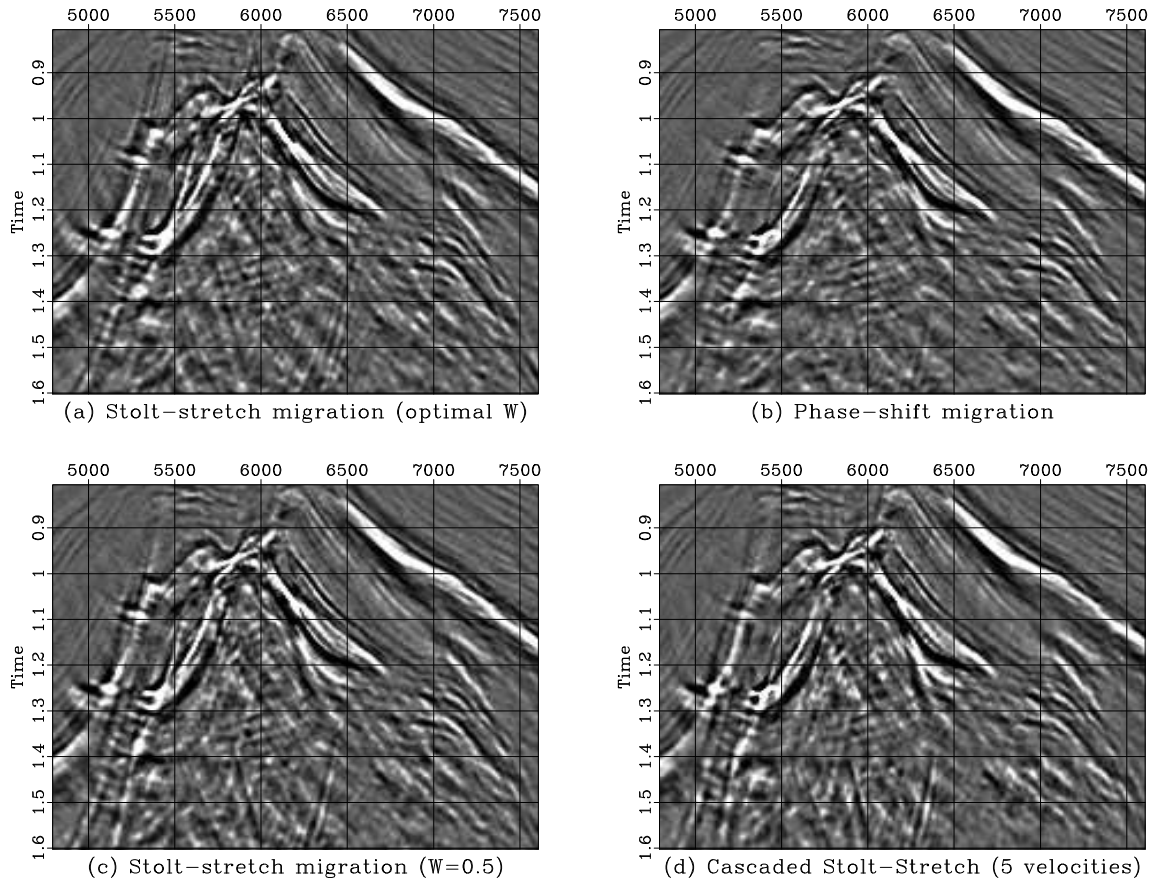


Figure 7: Zoom in the salt body area where steep dips are located. (a) Migration with the Stolt-stretch method using optimal W . (b) Migration with the phase-shift method. (c) Migration with the Stolt-stretch method using $W = 0.5$. (d) Migration with the cascaded Stolt-stretch approach, using 5 velocities.

REFERENCES

- Alkhalifah, T., and I. Tsvankin, 1995, Velocity analysis for transversely isotropic media: *Geophysics*, **60**, 1550–1566.
- Beasley, C., W. Lynn, K. Larner, and H. Nguyen, 1988, Cascaded frequency-wavenumber migration - Removing the restrictions on depth-varying velocity: *Geophysics*, **53**, 881–893.
- Castle, R. J., 1988, Shifted hyperbolas and normal moveout, *in* 58th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts: Soc. Expl. Geophys., Session: S9.3.
- Claerbout, J. F., 1985, *Imaging the Earth's Interior*: Blackwell Scientific Publications.
- de Bazelaire, E., 1988, Normal moveout revisited – inhomogeneous media and curved interfaces: *Geophysics*, **53**, 143–157.
- Fomel, S., 1997, Velocity continuation and the anatomy of prestack residual migration: 67th Ann. Internat. Mtg, Soc. of Expl. Geophys., 1762–1765.
- Fomel, S., and V. Grechka, 1996, On nonhyperbolic reflection moveout in anisotropic media, *in* SEP-92: Stanford Exploration Project, 135–158.
- Fomel, S. B., 1994, The method of velocity continuation in the seismic time migration problem: *Russian Geology and Geophysics*, **35**, 100–111.
- Gazdag, J., 1978, Wave equation migration with the phase-shift method: *Geophysics*, **43**, 1342–1351.
- Larner, K., and C. Beasley, 1987, Cascaded migrations - Improving the accuracy of finite-difference migration: *Geophysics*, **52**, 618–643. (Errata in GEO-52-8-1165).
- Larner, K. L., C. J. Beasley, and W. S. Lynn, 1989, In quest of the flank: *Geophysics*, **54**, 701–717. (Erratum in GEO-54-7-932-932; Discussion in GEO-54-12-1648-1650 with reply by authors).
- Levin, S., 1983, Remarks on two-pass 3-D migration error, *in* SEP-35: Stanford Exploration Project, 195–200.
- , 1985, Understanding Stolt stretch, *in* SEP-42: Stanford Exploration Project, 373–374.
- Malovichko, A. A., 1978, A new representation of the traveltime curve of reflected waves in horizontally layered media: *Applied Geophysics (in Russian)*, **91**, 47–53.
- Popovici, A. M., F. Muir, and P. Blondel, 1996, Stolt redux: A new interpolation method: *Journal of Seismic Exploration*, **5**, 341–347.
- Sava, P., 2000, Prestack Stolt residual migration for migration velocity analysis, *in* 70th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts: Soc. Expl. Geophys., 992–995.
- Stolt, R. H., 1978, Migration by Fourier transform: *Geophysics*, **43**, 23–48. (Discussion and reply in GEO-60-5-1583).
- , 1996, Short note - A prestack residual time migration operator: *Geophysics*, **61**, 605–607.
- Sword, C. H., 1987, A Soviet look at datum shift, *in* SEP-51: Stanford Exploration Project, 313–316.
- Thomsen, L., 1986, Weak elastic anisotropy: *Geophysics*, **51**, 1954–1966. (Discussion in GEO-53-04-0558-0560 with reply by author).

Yilmaz, Ö., 2001, Seismic data analysis: Processing, inversion, and interpretation of seismic data: Soc. Expl. Geophys.

Yilmaz, Ö., I. Tanir, C. Gregory, and F. Zhou, 2001, Interpretive imaging of seismic data: The Leading Edge, **20**.

APPENDIX A

In this Appendix, we derive an explicit expression for the Stolt-stretch parameter W by comparing the accuracy of equations (11) and (13), which approximate the travelttime curve in the neighborhood of the vertical ray. It is appropriate to consider a series expansion of the diffraction travelttime in the vicinity of the vertical ray:

$$t_0(l) = t_0|_{l=0} + \frac{1}{2} \left. \frac{d^2 t_0}{dl^2} \right|_{l=0} l^2 + \frac{1}{4!} \left. \frac{d^4 t_0}{dl^4} \right|_{l=0} l^4 + \dots, \quad (\text{A-1})$$

where $l = x - x_0$. Expansion (A-1) contains only even powers of l because of the obvious symmetry of t_0 as a function of l .

Matching the series expansions term by term is a constructive method for relating different equations to each other. The special choice of parameters t_v , v_{rms} , and S allows Malovichko's equation (13) to provide correct values for the first three terms of expansion (A-1):

$$t_0|_{l=0} = t_v; \quad (\text{A-2})$$

$$\left. \frac{d^2 t_0}{dl^2} \right|_{l=0} = \frac{1}{t_v v_{rms}^2(t_v)}; \quad (\text{A-3})$$

$$\left. \frac{d^4 t_0}{dl^4} \right|_{l=0} = -\frac{3S(t_v)}{t_v^3 v_{rms}^4(t_v)}. \quad (\text{A-4})$$

Considering Levin's equation (11) as an implicit definition of the function $t_0(t_v)$, we can iteratively differentiate it, following the rules of calculus:

$$\left. \frac{ds}{dl} \right|_{l=0} = s'(t_0) \left. \frac{dt_0}{dl} \right|_{l=0} = 0;$$

$$\left. \frac{d^2 s}{dl^2} \right|_{l=0} = \left(s'(t_0) \left. \frac{d^2 t_0}{dl^2} + s''(t_0) \left(\left. \frac{dt_0}{dl} \right)^2 \right) \right) \Big|_{l=0} = s'(t_v) \left. \frac{d^2 t_0}{dl^2} \right|_{l=0} = \frac{1}{v_0^2 s(t_v)}; \quad (\text{A-5})$$

$$\left. \frac{d^3 s}{dl^3} \right|_{l=0} = \left(3 s''(t_0) \frac{dt_0}{dl} \left. \frac{d^2 t_0}{dl^2} + s'(t_0) \left. \frac{d^3 t_0}{dl^3} + s'''(t_0) \left(\left. \frac{dt_0}{dl} \right)^3 \right) \right) \right) \Big|_{l=0} = 0$$

$$\begin{aligned}
\left. \frac{d^4 s}{dl^4} \right|_{l=0} &= \left(6 s'''(t_0) \left(\frac{dt_0}{dl} \right)^2 \frac{d^2 t_0}{dl^2} + 3 s''(t_0) \left(\frac{d^2 t_0}{dl^2} \right)^2 + 4 s''(t_0) \frac{dt_0}{dl} \frac{d^3 t_0}{dl^3} + \right. \\
&\quad \left. + s'(t_0) \frac{d^4 t_0}{dl^4} + s^{IV}(t_0) \left(\frac{dt_0}{dl} \right)^4 \right) \Big|_{l=0} = \\
&= \left(s''(t_v) \left(\frac{d^2 t_0}{dl^2} \right)^2 + s'(t_v) \frac{d^4 t_0}{dl^4} \right) \Big|_{l=0} = -\frac{3W}{v_0^4 s^3(t_0)} . \tag{A-6}
\end{aligned}$$

Substituting the definition of Stolt stretch transform (5) into (A-5) produces an equality similar to (A-3), which means that approximation (11) is theoretically accurate in depth-varying velocity media up to the second term in (A-1). It is this remarkable property that proves the validity of the Stolt stretch method (Levin, 1983; Claerbout, 1985). Moreover, equation (11) is accurate up to the third term if the value of the fourth-order traveltime derivative in (A-6) coincides with (A-4). Substituting equation (A-4) into (A-6) results in the expression

$$\frac{1 - W}{v_0^2 s^2(t_v)} = \frac{v^2(t_v) - S(t_v) v_{rms}^2(t_v)}{v_{rms}^4(t_v) t_v^2} . \tag{A-7}$$

It is now easy to derive from equation (A-7) the desired explicit expression for the Stolt stretch parameter W : equation (17) in the main text.