

Noncausal f - x - y regularized nonstationary prediction filtering for random noise attenuation on 3D seismic data^a

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ABSTRACT

Seismic noise attenuation is very important for seismic data analysis and interpretation, especially for 3D seismic data. In this paper, we propose a novel method for 3D seismic random noise attenuation by applying noncausal regularized nonstationary autoregression (NRNA) in f - x - y domain. The proposed method, 3D NRNA (f - x - y domain) is the extended version of 2D NRNA (f - x domain). f - x - y NRNA can adaptively estimate seismic events of which slopes vary in 3D space. The key idea of this paper is to consider that the central trace can be predicted by all around this trace from all directions in 3D seismic cube, while the 2D f - x NRNA just considers the middle trace can be predicted by adjacent traces along one space direction. 3D f - x - y NRNA uses more information from circumjacent traces than 2D f - x NRNA to estimate signals. Shaping regularization technology guarantees the nonstationary autoregression problem can be realizable in mathematics with high computational efficiency. Synthetic and field data examples demonstrate that, compared with f - x NRNA method, f - x - y NRNA can be more effective in suppressing random noise and improve trace-by-trace consistency, which are useful in conjunction with interactive interpretation and auto-picking tools such as automatic event tracking.

INTRODUCTION

Seismic noise attenuation is very important for seismic data processing and interpretation, especially for 3D seismic data. Among the methods of seismic noise attenuation, prediction filtering is one of the most effective and most commonly used methods, e.g., (Gulunay, 1986; Galbraith, 1984; Gulunay et al., 1993; Sacchi and Kuehl, 2001). Prediction filtering can be implemented in f - x domain or t - x domain (Hornbostel, 1991; Abma and Claerbout, 1995). Abma and Claerbout (1995) compared f - x method and t - x method and gave the advantages and disadvantages of both these methods. The proposed method in our paper belongs to the category of f - x domain methods. The f - x prediction technique was introduced for random noise attenuation on 2D poststack data by Canales (1984) and further developed by Gulunay (1986). Wang and West (1991) and Hornbostel (1991) used noncausal filters for

random noise attenuation on stacked seismic data and obtain a good result. Linear prediction filtering states that the signal can be described by an autoregressive (AR) model, which means that a superposition of linear events transforms into a superposition of complex sinusoids in the $f-x$ domain. Sacchi and Kuehl (2001) utilized the autoregressive-moving average (ARMA) structure of the signal to estimate a prediction error filter (PEF) and applied ARMA model to attenuate random noise. Liu et al. (2009) applied ARMA-based noncausal spatial prediction filtering to avoid the model inconsistency.

As already noted, these above mentioned $f-x$ methods assume seismic section is composed of a finite number of linear events with constant dip in $t-x$ domain. To cope with the assumption continuous changes dips, short temporal and spatial analysis windows are usually used in $f-x$ prediction filtering. Except using windowing strategy, several nonstationary prediction filters are proposed and used in seismic noise attenuation and interpolation. Naghizadeh and Sacchi (2009) proposed an adaptive $f-x$ prediction filter, which was used to interpolate waveforms that have spatially variant dips. Fomel (2009) developed a general method of regularized nonstationary autoregression (RNA) with shaping regularization (Fomel, 2007) for time domain inverse problems. Liu et al. (1991) propose a method for random noise attenuation in seismic data by applying noncausal regularized nonstationary autoregression (NRNA) in frequency domain, which is implemented for 2D seismic data. These nonstationary methods can control algorithms adaptability to changes in local dip so that they can process curved events.

If using $f-x$ prediction filter to suppress random noise on 3D seismic data, one need to run the 2D algorithm slice by slice (along inline x or crossline y). To use more information to predict the effective signal in 3D data, several geophysicists extended $f-x$ prediction filtering to 3D case. Chase (1992) designs and applies 2-D prediction filters in the plane defined by the inline and crossline directions for each temporal frequency slice of the 3-D data volume. Ozdemir et al. (1999) applied $f-x-y$ projection filtering to attenuate random noise of seismic data with low poor signal to noise ratio (SNR), in which the crucial step of 2-D spectral factorization is achieved through the causal helical filter. Gulunay (2000) proposed using full-plane noncausal prediction filters to process each frequency slice of the 3-D data. Wang (2002) applied $f-x-y$ 3D prediction filter to implement seismic data interpolation and gave a good result. Hodgson et al. (2002) presented a novel method of noise attenuation for 3D seismic data, which applies a smoothing filter to each targeted frequency slice and allows targeted filtering of selected parts of the frequency spectrum.

In this paper, we extend $f-x$ NRNA method (Liu et al., 1991) to $f-x-y$ case and use $f-x-y$ NRNA to attenuate random noise for 3D seismic data. The coefficients of 3D NRNA method are smooth along two space coordinates (x and y) in $f-x-y$ domain. This paper is organized as follows: First, we provide the theory for random noise on 3D seismic data, paying particular attention to establishment of $f-x-y$ NRNA equations with constraints and implementation of it with shaping regularization. Then we evaluate and compare the proposed method with $f-x$ NRNA using synthetic and

real data examples and discuss the parameter selection problem associated with our algorithm.

METHODOLOGY

The review of *f-x* NRNA

Seismic section $S(t, x)$ in *f-x* domain is predictable if it only includes linear events in $t - x$ domain. The relationship between the n -th trace and $(n-i)$ -th trace can be easily described as

$$S_n(f) = \sum_{i=1}^M a_i(f) S_{n-i}(f), \quad (1)$$

where M is the number of events in 2D seismic section. Eq. (1) describes forward prediction equations, namely causal prediction filtering equations (Gulunay, 2000). In the case of both forward and backward prediction equations (noncausal prediction filter), Eq. 1 can be written as (Spitz, 1991; Gulunay, 2000; Naghizadeh and Sacchi, 2009; Liu et al., 1991)

$$S_n(f) = \sum_{i=1}^M a_i S_{n-i}(f) + \sum_{i=-1}^{-M} a_i S_{n-i}(f), \quad (2)$$

where M is the parameter related to the number of events. Note that Eq. 2 implies the assumption $\sum_{i=1}^M a_i S_{n-i}(f) = 0.5 S_n(f)$ and $\sum_{i=-1}^{-M} a_i S_{n-i}(f) = 0.5 S_n(f)$. Theoretically, a_i in forward prediction equations is the complex conjugation of a_{-i} in backward equations (Galbraith, 1984). Gulunay (2000) pointed that it is possible to reduce the order of the normal equations from $2M$ to M because the coefficients of noncausal prediction filter have conjugate symmetry. *f-x* prediction filtering has the assumption that the events of seismic section are linear. If seismic events are not linear, or the amplitudes of wavelet are varying, they no longer follow linear or stationary assumptions (Canales, 1984). One needs to perform *f-x* prediction filtering over a short sliding window in time and space to cope with continuous changes in dips (Naghizadeh and Sacchi, 2009). Fomel (2009) developed a general method of RNA using shaping regularization technology, which is implemented for real number. Liu et al. (1991) extended the RNA method to *f-x* domain for complex numbers and applied it to seismic random noise attenuation for 2D seismic data. The *f-x* NRNA is defined as (Liu et al., 1991)

$$\varepsilon_n(f) = S_n(f) - \sum_{i=1}^M a_{n,i}(f) S_{n-i}(f) - \sum_{i=-1}^{-M} a_{n,i}(f) S_{n-i}(f). \quad (3)$$

Eq. 3 indicates that one trace noise-free in *f-x* domain can be predicted by adjacent traces with the different weights $a_{n,i}(f)$. Note that the weights $a_{n,i}(f)$ is

varying along the space direction, which indicated by subscript i in $a_{n,i}(f)$. In Eq. 3, the coefficients a_i is the function of space i , but it is not in Eq. 2. When applying $f-x$ NRNA to seismic noise attenuation, we assume the prediction error $\varepsilon_n(f)$ is the random noise and the predictable part $\sum_{i=1}^M a_{n,i}(f)S_{n-i}(f) + \sum_{i=-1}^{-M} a_{n,i}(f)S_{n-i}(f)$ is the signal. Finding spatial-varying coefficients $a_{n,i}(f)$ from Eq. 3 is ill-posed problem because there are more unknown variables than constraint equations. To obtain the coefficients, we should add constraint equations. Shaping regularization (Fomel, 2009) can be used to solve the under-constrained problem (Liu et al., 1991). The RNA method can also be used for seismic data processing in $t-x-y$ domain, such as seismic data interpolation (Liu and Fomel, 2011).

$f-x-y$ NRNA for random noise attenuation

Two dimensional $f-x$ NRNA only considers one space coordinate x . If we use $f-x$ NRNA on 3D seismic cube, we usually apply $f-x$ RNA in one space slice. $f-x$ NRNA reduces the effectiveness because the plane event in 3D cube is predictable along different directions rather than only one direction. Therefore, we should develop 3D $f-x-y$ NRNA to suppress random noise for 3D seismic data.

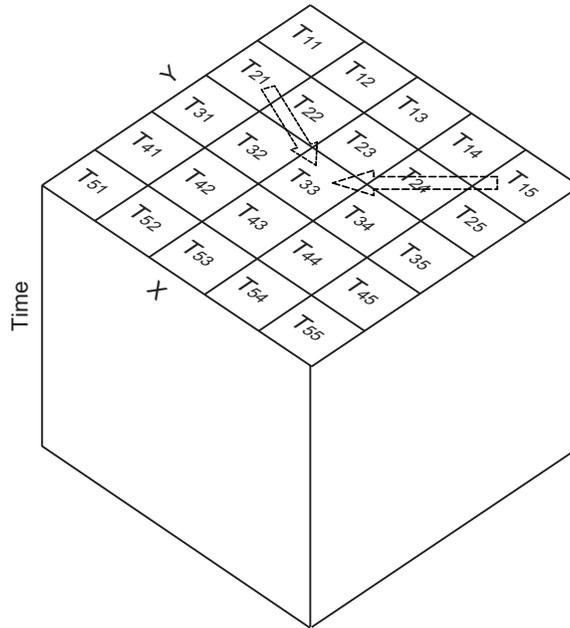


Figure 1: The $f-x-y$ prediction filter. The trace T_{33} is predicted from circumjacent traces $T_{11} \sim T_{55}$ (except itself T_{33}).

Next, we use Fig. 1 to illustrate the idea of $f-x-y$ NRNA. The middle trace T_{33} is the one we want to predict. Trace T_{33} can be predicted from circumjacent traces $T_{11} \sim T_{55}$ (except itself T_{33}). The prediction process includes all different directions. For example, if we use T_{21} to predict T_{33} , we can estimate a corresponding coefficient

using the described algorithm in the following. *f-x-y* NRNA uses all around traces to predict the middle trace. Therefore, the prediction uses more information than *f-x* NRNA. For all the traces in 3D cube, similar to the trace T_{33} , we can use circumjacent traces to predict them. Mathematically, we can write the prediction process as

$$S_{x,y}(f) = \sum_{i=-M, i \neq 0}^M a_i(f) S_{x,y,i}(f), \quad (4)$$

where M and i are the number and index of circumjacent traces, respectively. In the case of Fig. 1, $M=24$ and i is from 1 to 24. Note that $S_{x,y,i}(f)$ indicates the 24 circumjacent traces around $S_{x,y}(f)$. Eq. 4 is the equations of noncausal regularized stationary autoregression. Similarly to *f-x* NRNA, considering the nonstationary case, we can obtain

$$\tilde{S}_{x,y}(f) = \sum_{i=-M, i \neq 0}^M a_{x,y,i}(f) S_{x,y,i}(f), \quad (5)$$

where $a_{x,y,i}(f)$ is the space-varying coefficients, which means they have three free degrees, space axis x , space axis y and shift axis i . $\tilde{S}_{x,y}(f)$ can be regarded as the estimation of noise-free signal. However, the coefficients $a_{x,y,i}(f)$ are not known. Once we obtain the coefficients, we can estimate the effective signal using Eq. 5. Similar to *f-x* NRNA, we use shaping regularization to solve this ill-posed problem. Here, we assume that the coefficients $a_{x,y,i}(f)$ *f-x-y* RNA are smooth along two space axes x and y , which is reasonable because the curved surface event in 3D seismic data is locally plane. Therefore, we can obtain the following least square problem with shaping regularization

$$\min_{a_{x,y,i}(f)} \|S_{x,y}(f) - \sum_{i=-M, i \neq 0}^M a_{x,y,i}(f) S_{x,y,i}(f)\|_2^2 + R[a_{x,y,i}(f)], \quad (6)$$

where $R[\cdot]$ denotes shaping regularization term which constrains coefficients $a_{x,y,i}(f)$ to be smooth along space axes. We use one coefficient with a given frequency and a given shift (e.g., from T_{21} to T_{33} indicated by arrow in Fig. 1) to explain the constraint in Eq. 6. This 3D cube of coefficient with a given frequency and a given shift can be expressed as $a_{x,y,i_0}(f_0)$, which is smooth along with variables x and y . The smooth constraint of coefficients is the objective of shaping regularization. Finally, we use Eq. 6 to obtain the complex coefficients of *f-x-y* RNA, and use Eq. 5 to achieve the estimation of signal.

Transform-base methods can also be used for seismic noise attenuation (Ma and Plonka, 2010). Tang and Ma (1991) proposed to total-variation-based curvelet shrinkage for 3D seismic data denoising in order to suppress nonsmooth artifacts caused by the curvelet transform. Because the *f-x-y* NRNA method uses shaping regularization to solve the ill-posed inverse problem and is complemented in frequency domain, it has higher computation efficiency than curvelet-based methods.

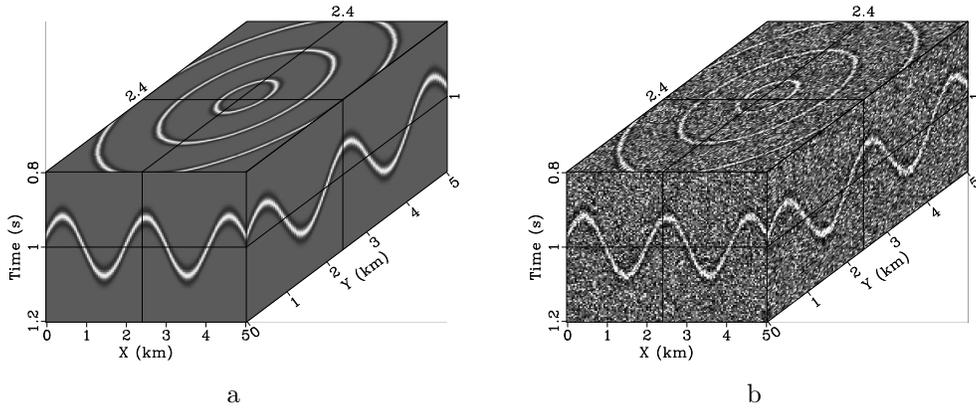


Figure 2: Synthetic benchmark 3D cube with one curved surface event (a) and noisy data cube (b).

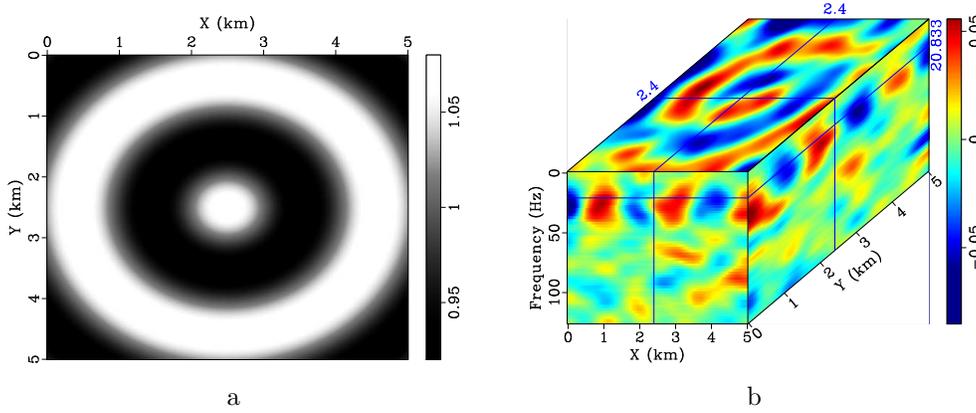


Figure 3: (a) Travel time of the event in Fig. 2a. (b) The imaginary part of $f-x-y$ NRNA coefficients at a given shift $a_{x,y,i_0}(f)$

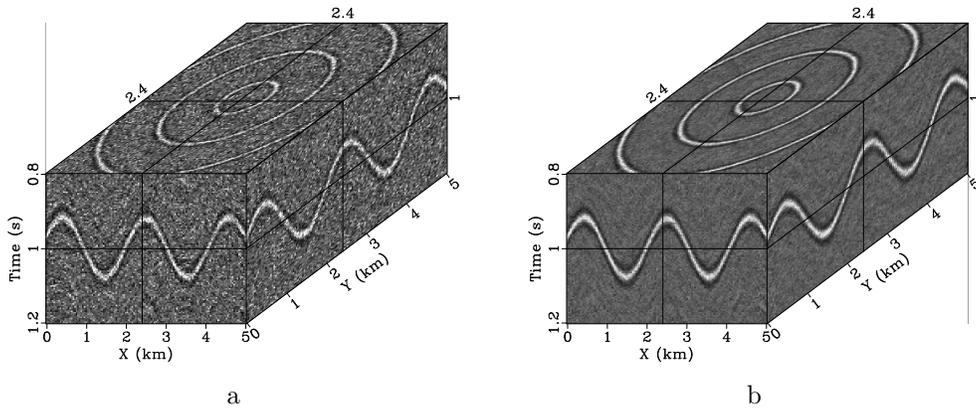


Figure 4: The results of $f-x$ NRNA (a) and $f-x-y$ NRNA (b).

SYNTHETIC EXAMPLES

We demonstrate the effectiveness of the proposed *f-x-y* NRNA using two synthetic datasets. The first synthetic example involves only one curved surface. Fig. 2a shows the synthetic dataset. Three slices of Fig. 2a illustrate the $Y=2.4$ km, $X=2.4$ km and $\text{Time}=1$ s, respectively. The following figures in this paper have the same way for display. The traveltimes of this surface is shifted sine function (Fig. 3a). We can find that the traveltimes are not linearly varying. Therefore, we cannot use stationary *f-x-y* prediction filtering to estimate the effective signal. Fig. 5b is the noisy data. This curved surface event is greatly contaminated by random noise. We respectively use *f-x* NRNA and *f-x-y* NRNA to attenuate the random noise and compare their results (Fig. 5c- 5d). The SNRs of *f-x* NRNA and *f-x-y* NRNA are 0.34 and 2.4, respectively. Although *f-x* NRNA has suppressed a lot of random noise, there are still some random noises in the result (Fig. 5c). Compared with *f-x* NRNA, *f-x-y* NRNA gives a better result. The curved surface event is very clear and consistent, which may be easier to automatically track for interpretation. Fig. 3b shows the imaginary part of *f-x-y* NRNA coefficients at a given shift $a_{x,y,i_0}(f)$, which are smooth along with space axes. From the slice with 20.833 Hz (up slice in Fig. 3b), one can find that the coefficients reflect the information of traveltimes or time shifts between circumjacent traces (Fig. 3a). From the frontal and lateral slices in Fig. 3b, one can conclude that the coefficients are related to dips of events from the frontal and lateral slices in Fig. 2a. The coefficient is zero if the event is horizontal (e.g. position B). The coefficients are respectively positive and negative if the events are upgoing (e.g. position A) and downgoing (e.g. position C). The estimated results of coefficients are consistent with theoretical analysis.

The second synthetic example is a synthetic shot gather with four hyperbolic events (Fig. 5a). Here, we consider anisotropy of the propagating velocity, so that there are intersecting events in Y slice but they are not intersecting in X slice (the second and third events). Comparing the results of *f-x* NRNA (Fig. 5c) and *f-x-y* NRNA (Fig. 5d), we can find that *f-x-y* NRNA can remove more noise than *f-x* NRNA, especially for poor signals (for example, far offset of the events indicated by arrows in Fig. 5a- 5d). The SNRs of *f-x* NRNA and *f-x-y* NRNA are 0.95 and 2.61, respectively. Both of these synthetic examples demonstrate that the proposed *f-x-y* NRNA can be effectively used to attenuate random noise for 3D seismic data cubes.

APPLICATION ON FIELD POSTSTACK DATA

The *f-x-y* NRNA method is applied to a 3D image after time migration (Fig. 6). The shallow structures are simple plane layers (above 1 s) and the deep structures are complex curved layers (below 1 s). We respectively apply *f-x* NRNA and *f-x-y* NRNA to enhance the reflectors of this 3D image cube. Fig. 7 shows the imaginary part of *f-x-y* NRNA coefficients at a given shift $a_{x,y,i_0}(f)$. Similar to the synthetic example, the *f-x-y* NRNA coefficients are smooth and reflect the information of event dips. In

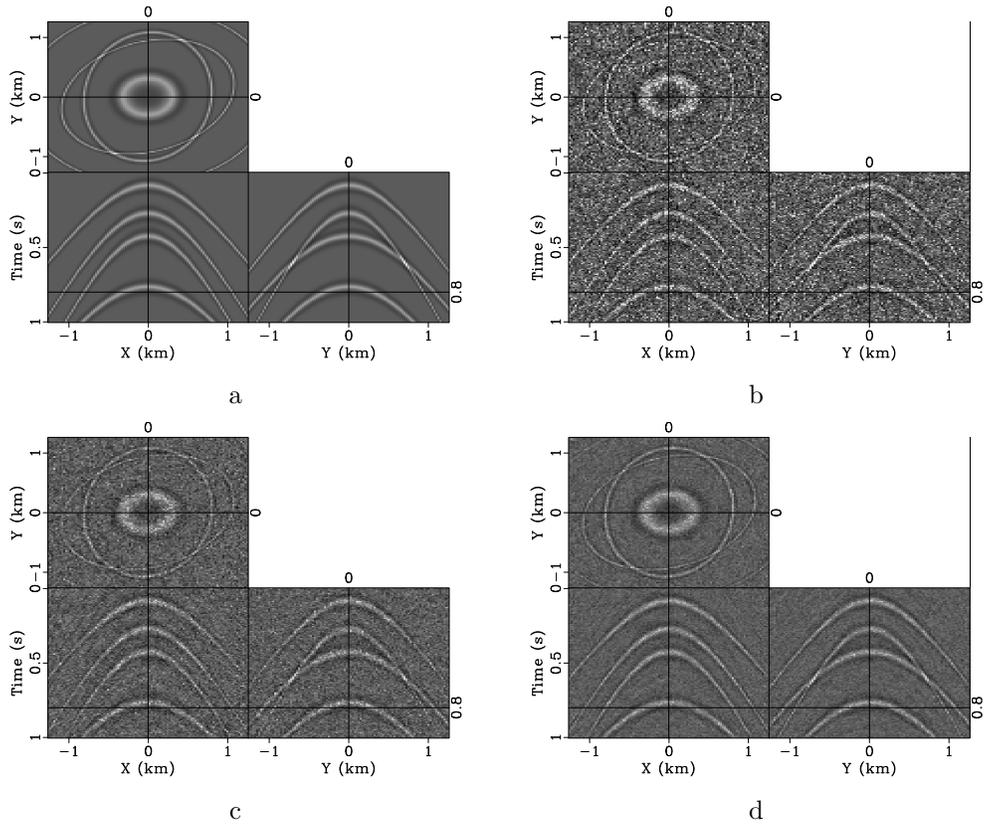


Figure 5: (a) Synthetic 3D shot gather. (b) Noisy data. (c) The result of $f-x$ RNA. (d) The result of $f-x-y$ RNA.

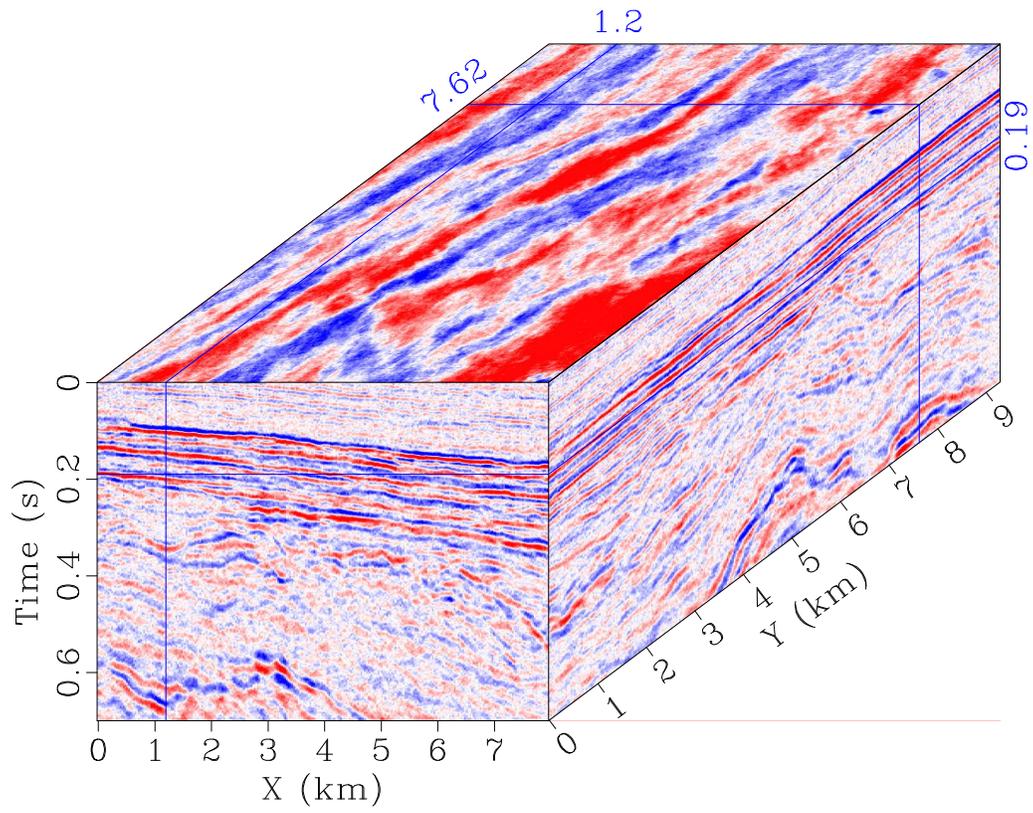


Figure 6: The 3D field data cube after time migration.

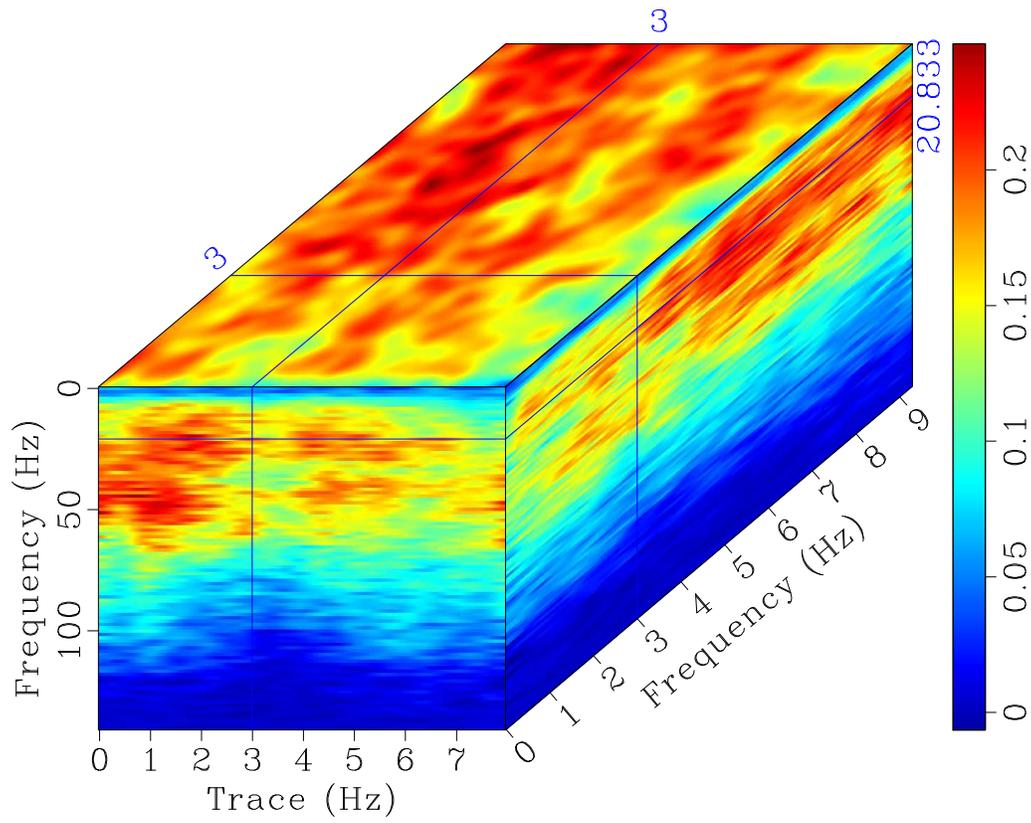


Figure 7: The imaginary part of f - x - y NRNA coefficients at a given shift $a_{x,y,i_0}(f)$ for real dataset.

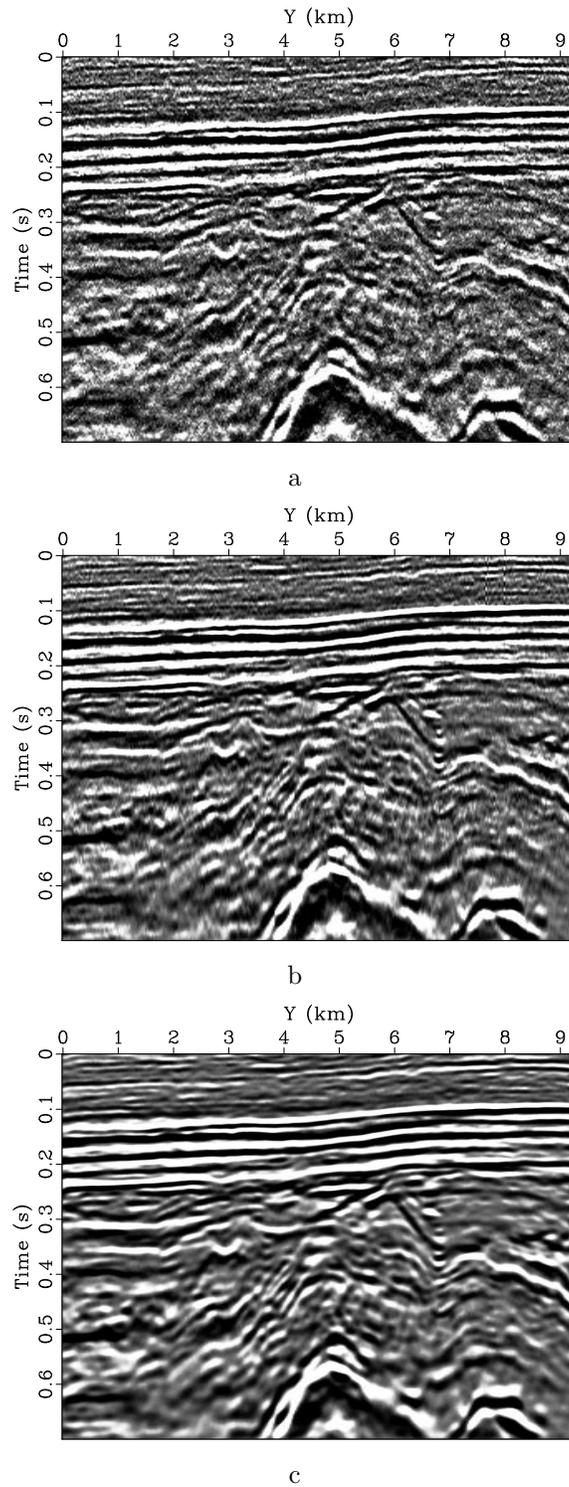
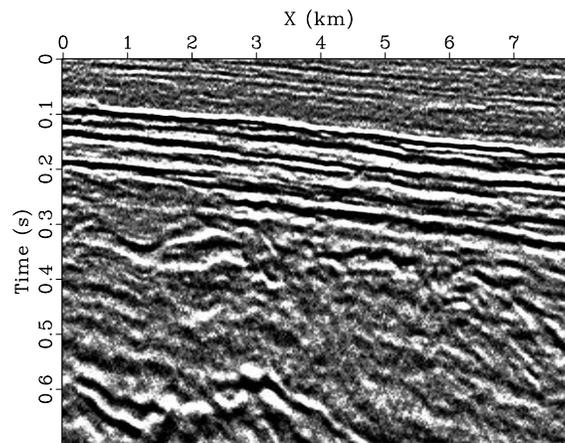
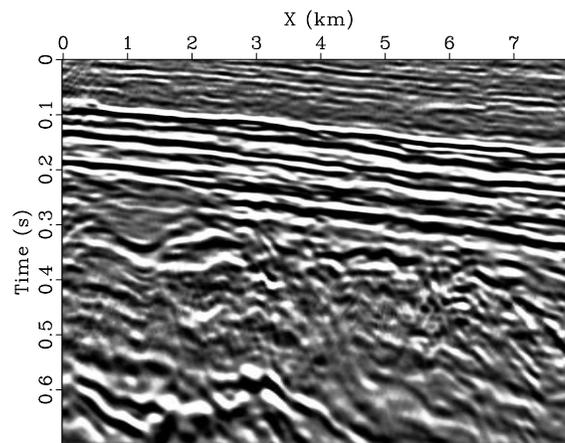


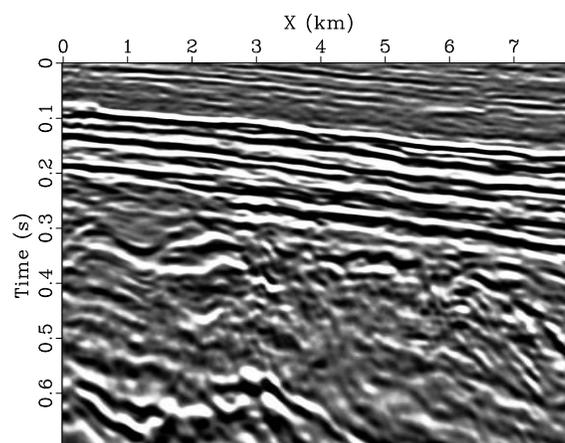
Figure 8: The slice X of field data cube. (a) Original data; (b) $f-x$ NRNA; (c) $f-x-y$ NRNA.



a



b



c

Figure 9: The slice Y of field data cube. (a) Original data; (b) f - x NRNA; (c) f - x - y NRNA.

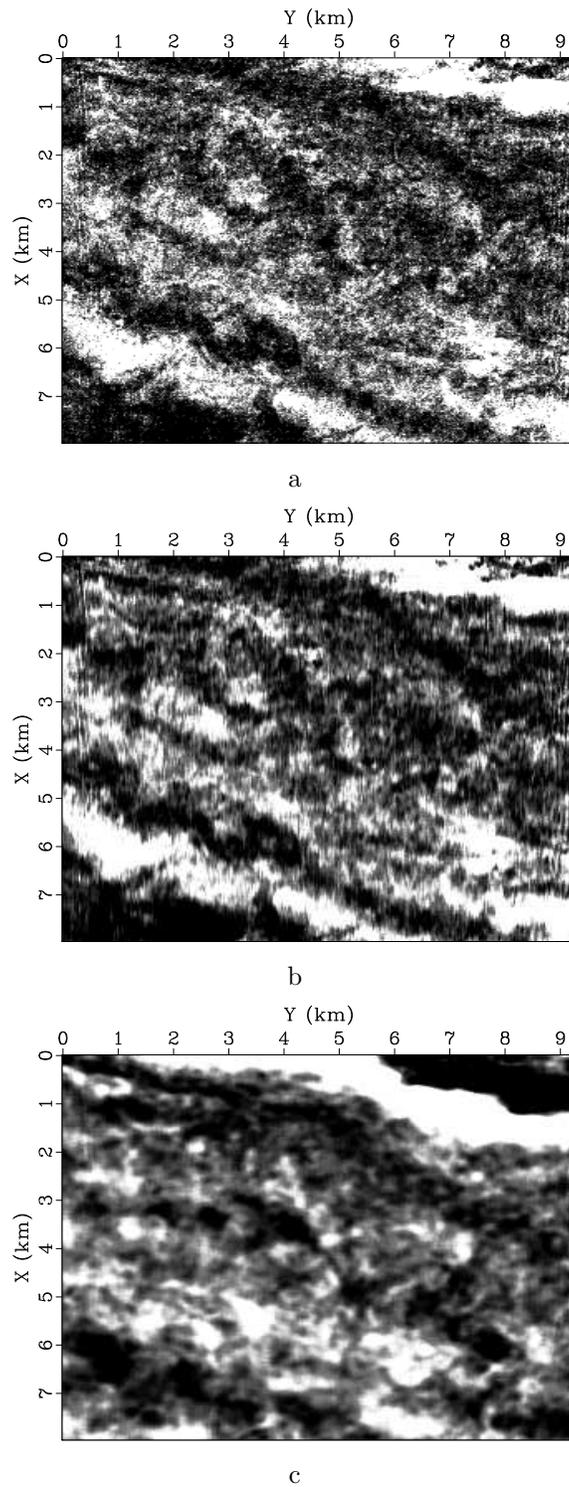


Figure 10: The time slice of field data cube. (a) Original data; (b) $f-x$ NRNA; (c) $f-x-y$ NRNA.

this example, we use $M=2$ for $f-x-y$ NRNA and $M=8$ for $f-x$ NRNA, respectively. Figs. 8a- 8c and 9a- 9c respectively shows the X and Y slices after $f-x$ NRNA noise attenuation and $f-x-y$ NRNA noise attenuation. We can find that $f-x-y$ NRNA method can give a better result than $f-x$ NRNA method. The result of $f-x-y$ NRNA has a much better lateral continuity. These two methods not only improve the shallow plane events evidently (e.g. 0s -0.5s), but also improve the deep curved surface events (e.g. the area indicated by ellipse). This is because these two methods both are nonstationary methods, which is suitable for curved events. In addition, comparing $f-x$ NRNA and $f-x-y$ NRNA methods from time slices (Fig. 10a- 10c), one can also see that the $f-x-y$ NRNA gives more consistent result. The lateral continuity and trace-by-trace consistency of the reflections are crucial in structural interpretation of seismic data by reflection picking especially for the auto-picking tools of interactive interpretation systems (Fomel, 2010).

CONCLUSIONS

We have proposed a novel method for seismic noise attenuation using $f-x-y$ NRNA for 3D seismic data. $f-x-y$ NRNA is the 3D extension of $f-x$ NRNA. By using more information to predict the seismic signal, the $f-x-y$ NRNA improves the denoising result for 3D seismic data. The varying coefficients of the $f-x-y$ NRNA are smooth along space coordinates for a given direction. The smoothness is controlled by shaping regularization, which has the key parameter: the smooth radius. The smooth radius can be selected by user according to the smoothness of assumed coefficients. This approach does not require breaking the input data into local windows along space axis, although it is conceptually analogous to sliding spatial windows with maximum overlap. Execution time of $f-x-y$ NRNA is reduced by iteration inversion and shaping regularization. Synthetic and field data examples both confirm that the proposed $f-x-y$ RNA approach can be significantly more effective in noise attenuation and consistency improvement than $f-x$ RNA for 3D seismic data. Therefore, it may be useful in conjunction with interactive interpretation systems and auto-picking tools such as automatic event tracking.

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