

# Continuous time-varying Q-factor estimation method in the time-frequency domain<sup>a</sup>

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## ABSTRACT

The Q-factor is an important physical parameter for characterizing the absorption and attenuation of seismic waves propagating in underground media, which is of great significance for improving the resolution of seismic data, oil and gas detection, and reservoir description. In this paper, the local centroid frequency is defined using shaping regularization and used to estimate the Q values of the formation. We propose a continuous time-varying Q-estimation method in the time-frequency domain according to the local centroid frequency, namely, the local centroid frequency shift (LCFS) method. This method can reasonably reduce the calculation error caused by the low accuracy of the time picking of the target formation in the traditional methods. The theoretical and real seismic data processing results show that the time-varying Q values can be accurately estimated using the LCFS method. Compared with the traditional Q-estimation methods, this method does not need to extract the top and bottom interfaces of the target formation; it can also obtain relatively reasonable Q values when there is no effective frequency spectrum information. Simultaneously, a reasonable inverse Q filtering result can be obtained using the continuous time-varying Q values.

## INTRODUCTION

The viscoelasticity and heterogeneity of subsurface media result in amplitude attenuation and phase distortion of seismic waves propagating in the media, considerably reducing the seismic data resolution. The study of the absorption and attenuation characteristics of underground media is of great significance to the inversion of geophysical properties and the distribution analysis of oil and gas reservoirs. The Q-factor is an important parameter for describing the above characteristics that reflect the formation structure and the comprehensive effect of fluid properties such as saturation, porosity, and permeability (Winkler and Nur, 1982). The Q-estimation methods are implemented in the time domain, frequency domain, or time-frequency domain.

The time-domain methods mainly include the amplitude decay method, the rise-time method (Kjartansson, 1979), the wavelet modeling method (Jannsen et al.,

1985), and the analytic signal method (Engelhard, 1996). The signal-to-noise ratio of acquired seismic data is low and the amplitude information is unreliable because of the influence of several environmental factors and complex medium structure. Thus, the accuracy and stability of  $Q$  values estimated in the time domain often cannot meet actual production requirements. Frequency information is relatively less affected by noise and data amplitude, so its stability is generally better than that of the time-domain method (Gong et al., 2009; Yan et al., 2001). The frequency domain methods include the spectral ratio (SR) method (Bath, 1974), the matching method (Raikes and White, 1984), the spectral modeling method (Jannsen et al., 1985), etc. The SR method is a widely used method with high accuracy in estimating the  $Q$  values without noise; however, it is greatly affected by the selected spectral bandwidth and noise (Tonn, 1991). Considering these factors, some scholars improved the SR method. Wang et al. (2015a) calculated the  $Q$  values using the logarithmic spectral area difference (LSAD) method, and Liu et al. (2018) introduced frequency-based linear fitting in the SR method. Quan and Harris (1997) proposed the centroid frequency shift (CFS) method based on the fact that the centroid frequency experiences a downshift when seismic waves propagate in viscoelastic media. This method deduces the relationship between the  $Q$ -factor and the centroid frequency when the amplitude spectrum shape is Gaussian, rectangular, or triangular. However, these are different from the shape of the actual seismic amplitude spectrum, so the CFS method may produce systematic errors. Some scholars analyzed and improved the CFS method, such as the  $Q$ -estimation method based on the combinations of statistical frequency attributes (Zhao et al., 2013) and the  $Q$ -value inversion using the centroid frequency of the energy spectrum (Wang et al., 2015b). Both techniques avoid the assumption of the amplitude spectrum of seismic wavelets. Zhang and Ulrych (2002) proposed the peak frequency shift (PFS) method to estimate the  $Q$  values of common midpoint (CMP) gathers, and Gao and Yang (2007) used this method to estimate the  $Q$  values of vertical seismic profiling (VSP) data. Li and Liu (2015) fitted the amplitude spectrum using the frequency-weighted-exponential function and derived a formula for estimating the  $Q$  value. Hu et al. (2013) established the relationship between the centroid and peak frequencies under the assumption of the Ricker wavelet and proposed an improved frequency shift (IFS) method. Li et al. (2015) estimated the  $Q$  values using the dominant and centroid frequencies. These frequency-domain methods must use the Fourier transform to calculate the frequency spectrum of the seismic signal in the selected time window. However, the actual seismic reflection data interfere with each other, causing some difficulties in selecting the appropriate type and length of the time window function. Furthermore, the frequency spectrum obtained using the Fourier transform reflects the average effect of a certain frequency component, reducing calculation accuracy of the  $Q$ -factors.

Several scholars calculate the  $Q$  values in the time-frequency domain to avoid selecting time windows and reduce the influence of spectral interference between adjacent seismic reflection waves. Reine et al. (2009) compared the  $Q$ -factors estimated using the SR method in four different time-frequency transform domains and concluded that the time-frequency transform with the variable window could achieve more accurate

and stable calculation results. Wang (2004) transformed poststack seismic data from the time domain to the time-frequency domain using the Gabor transform and then calculated the  $Q$  values according to the one-dimensional function of the product of frequency and time. Lupinacci and Oliveira (2015) estimated the  $Q$ -factors of the poststack seismic section in the Gabor transform domain using three different methods. An (2015) used the energy density ration method to estimate the  $Q$  values of CMP gathers based on an improved S-transform. Hao et al. (2016) derived the  $Q$ -estimation formula of the SR method in the generalized S-transform domain. Liu et al. (2011) improved the division operation of the SR method using shaping regularization and proposed a stable  $Q$ -estimation method based on the S-transform. Both Liu et al. (2016a) and Liu et al. (2016b) estimated the  $Q$ -factors based on the local time-frequency transform (LTFT). Wu et al. (2018) proposed a continuous spectral ratio slope method based on the generalized S-transform to avoid the extraction of target layers. Because the time-frequency transform can characterize the local attributes of the seismic data, this paper develops a continuous time-varying  $Q$ -estimation method in the time-frequency domain. The LTFT is an adaptive time-frequency transform proposed by Liu and Fomel (2013), based on the Fourier transform and solves the underdetermined problem of the least-squares solution of the adaptive Fourier series through shaping regularization (Fomel, 2007b). LTFT can adjust the frequency range and frequency sampling interval while providing variable resolution in the time direction.

When estimating the  $Q$  value in the frequency domain, accurately extracting the instantaneous spectrum of the target layers top and bottom interfaces is necessary. However, this is often difficult to achieve in actual data processing. This paper proposes a continuous time-varying  $Q$ -estimation method in the time-frequency domain based on the local centroid frequency to avoid picking up the target layer. The local centroid frequency is obtained by solving the inversion problem of the centroid frequency in the time-frequency domain within the constraint of the shaping regularization condition (Fomel, 2007b). This frequency is not calculated according to the instantaneous spectrum information at a specific moment but by adjusting the smoothing parameter to use the spectrum information around the moment. Therefore, we can get a relatively stable local centroid frequency at times without effective spectrum information. In this paper, the local centroid frequency and CFS method are combined to calculate the timevarying equivalent  $Q$ -factors and the time-varying interval  $Q$ -factors. Furthermore, we implement inverse  $Q$  filtering processing (Wang, 2002, 2006) using the continuous time-varying equivalent  $Q$ -factors.

## THEORY

### Local centroid frequency

The centroid frequency  $f_c$  with respect to the amplitude spectrum  $F(f)$  can be defined as (Quan and Harris, 1997)

$$f_c = \frac{\int_0^\infty f F(f) df}{\int_0^\infty F(f) df}, \quad (1)$$

where  $F(f)$  is the Fourier amplitude spectrum of the signal, and the centroid frequency  $f_c$  represents the first moment along the frequency direction of the amplitude spectrum.

The variance  $\sigma_c^2$  of the centroid frequency can be defined as

$$\sigma_c^2 = \frac{\int_0^\infty (f - f_c)^2 F(f) df}{\int_0^\infty F(f) df}, \quad (2)$$

The time-frequency spectrum  $B(f, t)$  replaces the Fourier amplitude spectrum  $F(f)$  in equations 1 and equations 2, and the instantaneous centroid frequency  $f_c(t)$  and instantaneous variance  $\sigma_c^2(t)$  of the amplitude spectrum are defined as

$$f_c(t) = \frac{\int_0^\infty f B(f, t) df}{\int_0^\infty B(f, t) df}, \quad (3)$$

$$\sigma_c^2(t) = \frac{\int_0^\infty (f - f_c(t))^2 B(f, t) df}{\int_0^\infty B(f, t) df}. \quad (4)$$

The above equations show that the instantaneous centroid frequency and variance are calculated instantaneously using the amplitude spectrum information at a specific time. However, at times without effective spectrum information, reasonable results cannot be obtained using this calculation method.

Fomel (2007a) defined the local attributes of the seismic signals such as local frequency and local similarity using shaping regularization. In this paper, we use a similar method to define the local centroid frequency and local variance. Equation 3 shows that the instantaneous centroid frequency is a division regarding two integrals and can be expressed in linear algebraic notation as

$$\mathbf{f} = \mathbf{L}^{-1} \mathbf{n}, \quad (5)$$

where  $\mathbf{n} = [\int_0^\infty f B(f, t_0) df \cdots \int_0^\infty f B(f, t_n) df]^T$  represents the vector of the numerator in equation 3,  $\mathbf{f}$  represents the vector of the instantaneous centroid frequency

$f_c(t)$ , and  $\mathbf{L} = \begin{bmatrix} \int_0^\infty B(f, t_0) df & 0 & \cdots & 0 \\ 0 & \int_0^\infty B(f, t_1) df & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \int_0^\infty B(f, t_n) df \end{bmatrix}$  is a diagonal matrix composed of the denominators in equation 3. Equation 5 can be regarded as an

inversion problem. We use the least-squares criterion to calculate  $\mathbf{f}$  to meet the objective function  $\min \|\mathbf{n} - \mathbf{L}\mathbf{f}\|_2^2$ , where  $\|\cdot\|_2^2$  represents the  $l_2$  norm. The regularization condition is added to constrain the inversion problem to satisfy the local property. For example, the traditional Tikhonov regularization constrains the model  $\mathbf{f}$  to satisfy the local smoothness, and the objective function becomes

$$\begin{cases} \mathbf{L}\mathbf{f} = \mathbf{n} \\ \varepsilon\mathbf{D}\mathbf{f} \approx 0 \end{cases}, \quad (6)$$

where  $\varepsilon$  controls the weight of the regularization term and  $\mathbf{D}$  is the Tikhonov regularization operator. In this case, the least-squares solution of the objective function under the regularization constraint is the local centroid frequency

$$\mathbf{f}_{loc} = (\mathbf{L}^T\mathbf{L} + \varepsilon^2\mathbf{D}^T\mathbf{D})^{-1}\mathbf{L}^T\mathbf{n}. \quad (7)$$

The theory of shaping regularization comes from data smoothing. It has fewer parameters and a faster convergence speed than the traditional Tikhonov regularization method. When considering shaping regularization, the shaping operator  $\mathbf{S}$  can be defined as

$$\mathbf{S} = (\mathbf{I} + \varepsilon^2\mathbf{D}^T\mathbf{D})^{-1}, \quad (8)$$

and,

$$\varepsilon^2\mathbf{D}^T\mathbf{D} = \mathbf{S}^{-1}\mathbf{I}. \quad (9)$$

The least-squares solution under the shaping regularization constraint can be obtained by substituting the above equation into equation 7

$$\mathbf{f}_{loc} = (\mathbf{L}^T\mathbf{L} + \mathbf{S}^{-1} - \mathbf{I})^{-1}\mathbf{L}^T\mathbf{n} = [\mathbf{I} + \mathbf{S}(\mathbf{L}^T\mathbf{L} - \mathbf{I})]^{-1}\mathbf{S}\mathbf{L}^T\mathbf{n}, \quad (10)$$

when the iterative algorithm is used to solve the above problem, the scale parameter  $\lambda$  can be introduced to improve the convergence speed and preserve physical dimensionality. Usually,  $\lambda$  is chosen as the least-squares norm of  $\mathbf{L}$  (Fomel, 2007a), and the local centroid frequency  $\mathbf{f}_{loc}$  is

$$\mathbf{f}_{loc} = [\lambda^2\mathbf{I} + \mathbf{S}(\mathbf{L}^T\mathbf{L} - \lambda^2\mathbf{I})]^{-1}\mathbf{S}\mathbf{L}^T\mathbf{n}. \quad (11)$$

Similarly, the local variance  $\sigma_{loc}^2$  can be calculated using the above method. When calculating the local centroid frequency, only one smoothing parameter is needed to control the locality and smoothness of the local centroid frequency. The local centroid frequency is not calculated instantaneously using the information at a specific time or calculated globally in a time window but is calculated locally using the information around the time. Thus, a relatively reasonable local centroid frequency can be continuously and smoothly calculated at the time of missing information (such as when the amplitude spectrum is zero). In this paper, we use the local centroid frequency to estimate the continuous time-varying  $Q$  values of the formation.

## Local centroid frequency shift (LCFS) method

Seismic waves propagating in underground media experience amplitude attenuation and phase distortion. High-frequency components attenuate faster than low-frequency components, so the centroid frequency of the amplitude spectrum experiences a down-shift in the propagation process. Quan and Harris (1997) proposed the CFS method according to the above phenomenon.

When considering seismic wave propagation in the viscoelastic medium, the amplitude spectrum of seismic waves with different travel times can be approximately expressed as (Zhang and Ulrych, 2002)

$$B(f, t) = A(t)B(f, t_0) \exp\left(-\frac{\pi f \Delta t}{Q}\right), \quad (12)$$

where  $Q$  is the quality factor,  $\Delta t = t - t_0$  is the travel time difference,  $A(t)$  is a frequency-independent factor (including spherical diffusion, transmission loss, etc.),  $B(f, t_0)$  is the seismic amplitude spectrum at time  $t_0$ , and  $B(f, t)$  is the amplitude spectrum at travel time  $t$ .

The CFS method assumes that the amplitude spectrum of the source wavelet satisfies the Gaussian distribution and can be expressed as

$$B(f, t_0) = \exp\left(-\frac{(f - f_c(t_0))^2}{2\sigma_c^2(t_0)}\right), \quad (13)$$

where  $f_c(t_0)$  and  $\sigma_c^2(t_0)$  represent the instantaneous centroid frequency and instantaneous variance of the amplitude spectrum at time  $t_0$ , respectively. The timevarying  $Q$ -factors estimated using the CFS method can be obtained from equations 3, 4, and 12:

$$Q(t) = \frac{\pi\sigma_c^2(t_0)(t - t_0)}{f_c(t_0) - f_c(t)}. \quad (14)$$

By replacing the instantaneous centroid frequency and instantaneous variance in equation 14 with the local centroid frequency and local variance, the time-varying  $Q$ -estimation equation can be rewritten as

$$Q_{loc}(t) = \frac{\pi\sigma_{loc}^2(t_0)(t - t_0)}{f_{loc}(t_0) - f_{loc}(t)}. \quad (15)$$

where  $f_{loc}(t_0)$  and  $\sigma_{loc}^2(t_0)$  represent the local centroid frequency and local variance of the amplitude spectrum at time  $t_0$ , respectively, and  $f_{loc}(t)$  is the local centroid frequency of the amplitude spectrum at time  $t$ . The above method of estimating the  $Q$  values using the local centroid frequency is called the LCFS. It can be seen from the equation that this method must estimate the  $Q$  value in the time-frequency domain.

The CFS method assumes that the amplitude spectrum of the source wavelet is Gaussian spectrum and that the variance of the amplitude spectrum does not

change with the attenuation effect. However, the amplitude spectrum of the actual seismic wave usually does not satisfy the Gaussian distribution. The absorption and attenuation effect would make the variance smaller and the bandwidth narrower, so the CFS method would produce the systematic error proportional to the travel time difference  $\Delta t$ . When the travel time difference of the two reflected waves is small, the variances of the two waves are approximately equal. Thus, this paper improves the Q-estimation accuracy by reducing the travel time difference. Assuming that each time sampling point corresponds to a stratum interface, the above equation can be used to calculate the interval Q-factors between every two adjacent time sampling points. Then, the interval Q-factors can be used to further estimate the equivalent Q-factors between the reference and the target layers. The amplitude spectrum of layer  $n$  can be expressed as (Zhang and Ulrych, 2002)

$$B(f, t_n) = A(t_n)B(f, t_0) \exp(-\pi f \sum_{i=1}^n \frac{\Delta t_i}{Q_i}), i = 1, 2, \dots, n, \quad (16)$$

where  $\Delta t_i = t_i - t_{i-1}$  and  $Q_i$  are the travel time and quality factor in layer  $i$ , respectively.

The above equation can be expressed by the equivalent Q theory as

$$\exp(-\pi f \frac{t_n}{Q_{n,eff}}) = \exp(-\pi f \sum_{i=1}^n \frac{\Delta t_i}{Q_i}), \quad (17)$$

The above equation can be simplified to

$$Q_{n,eff} = \frac{t_n}{\sum_{i=1}^n \frac{\Delta t_i}{Q_i}}, \quad (18)$$

where  $t_n = \sum_{i=1}^n \Delta t_i$  represents the total travel time of a reflection.

By substituting the equation of interval Q-factors estimated using the LCFS method into the above equation, the equivalent Q-factor of layer  $n$  ( $n$ th time sampling point) can be expressed as

$$Q_{n,eff} = \frac{t_n}{\sum_{i=1}^n \frac{\Delta t_i}{\frac{\pi \sigma_{loc}^2(t_{i-1}) \Delta t_i}{f_{loc}(t_{i-1}) - f_{loc}(t_i)}}}} = \frac{\pi t_n}{\sum_{i=1}^n \frac{f_{loc}(t_{i-1}) - f_{loc}(t_i)}{\sigma_{loc}^2(t_{i-1})}} \quad (19)$$

where  $Q_{n,eff}$  represents the equivalent Q-factor from the reference layer to layer  $n$  estimated using the LCFS method.

## SYNTHETIC DATA PROCESSING

### Estimation of local centroid frequency

To verify the feasibility of calculating the local centroid frequency using the LTFT method, a nonstationary signal (Figure 1b) is generated by convolving the Ricker

wavelet with a random reflection coefficient (Figure 1a). The dominant frequency of the signal is a function varying with time  $f_d(t) = 100 - 75t^2$ . Figure 2a shows a time-frequency spectrum consisting of the Ricker wavelets frequency spectrum (the dominant frequency is  $f_d(t) = 100 - 75t^2$ ). According to the dominant frequency of the Ricker wavelet, we can calculate the theoretical centroid frequency (black line in Figure 2a) of the Ricker wavelet  $f_c(t) = 2f_d(t)/\sqrt{\pi}$  (Hu et al., 2013). Figures 2b and 2c show the time-frequency spectrum of the synthetic signal obtained using the LTFT and S-transform, respectively. We estimated the local centroid frequency from these two time-frequency spectra, as shown in Figure 3 (the blue line is estimated from the LTFT, and the purple line is estimated from the S-transform). Compared with the theoretical centroid frequency (black line in Figures 2a and 3), the local centroid frequency obtained using the LTFT method is closer to the theoretical curve, so the LTFT analysis method is selected for calculating the local centroid frequency and time-varying Q-factors.

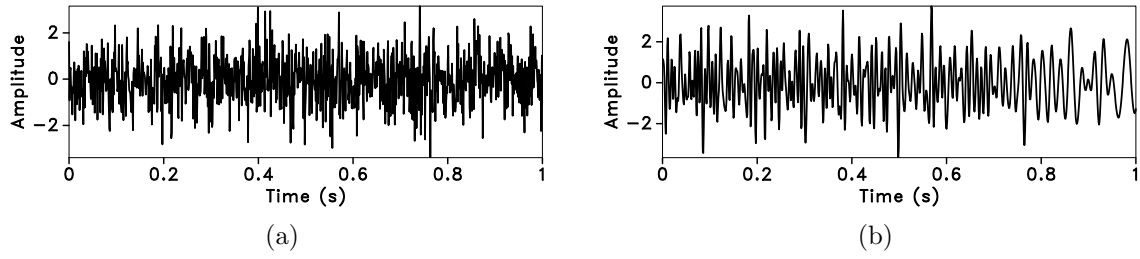


Figure 1: Theoretical model. Random reflectivity series (a), synthetic nonstationary signal (b).

## Attenuated model with constant Q-factors

Although the CFS and LCFS methods assume that the amplitude spectrum is Gaussian shape, they are still applicable to other spectra that fit well to the Gaussian spectrum, such as the Ricker wavelet (Quan and Harris, 1997). The solid line in Figure 4 is the amplitude spectrum of the Ricker wavelet (the dominant frequency is 50 Hz). According to the centroid frequency ( $f_c = 56.42$  Hz) and variance ( $\sigma_{loc}^2 = 566.9$ ) of the amplitude spectrum, the Gaussian spectrum can be synthesized (dotted line in Figure 4). The two amplitude spectra can be well fitted. In this paper, the Ricker wavelet is selected as the source wavelet to verify the effectiveness of the LCFS method in estimating the Q-factors. The Ricker wavelet with the dominant frequency of 50 Hz is used as the source wavelet to convolute with the reflectivity series to generate the attenuation model (Figure 5a). The time sampling interval is 1 ms, and the maximum propagation time is 1 s. There is one reflection interface at 0.1, 0.2, 0.4, 0.6, 0.7, and 0.9 s respectively, and the Q-factor of the formation is set to a constant value of 60, wherein the first layer is set to be unattenuated. Figure 5b shows the time-frequency spectrum of the attenuated signal obtained using the LTFT method. The LTFT method can accurately represent the time-frequency character-



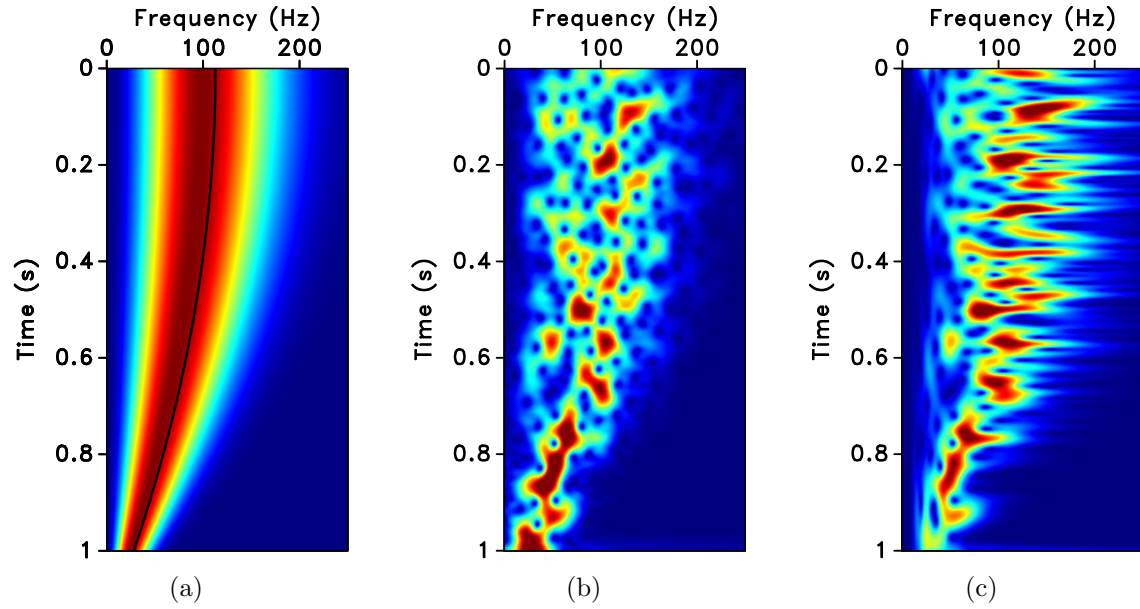


Figure 2: Time-frequency spectrum. Theoretical time-frequency spectrum (the black line represents the theoretical centroid frequency) (a), time-frequency spectrum of the LTFT (b), time-frequency spectrum of the S-transform (c).

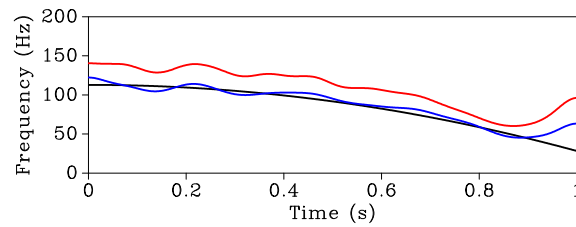


Figure 3: Local centroid frequency estimation (the black line represents the theoretical centroid frequency, the blue line is estimated using the LTFT method, and the purple line is estimated using the S-transform).

istics, wherein the black line is the local centroid frequency estimated using shaping regularization. By taking the local centroid frequency at 0.1 s as the reference value, the time-varying Q-value curve calculated using equation 19 is shown as the red line in Figure 5c. The black dotted line is the theoretical Q value. We can see that the time-varying Q value estimated using the proposed method is close to the theoretical Q value. Inverse Q filtering is performed using the estimated time-varying Q-factors, and the result is shown in Figure 5d. It can be seen that the amplitude and phase of the wavelets are well recovered. Figure 6 is the result estimated using the proposed method in the S-transform domain. Figure 6a is the time-frequency spectrum of the S-transform, and the black line is the local centroid frequency. Figure 6b shows the time-varying Q values estimated using the LCFS method, where the estimated result before 0.6 s has a large error relative to the result estimated using the LTFT method, and the estimated result after 0.6 s is close to the theoretical one. Figure 6c shows the inverse Q filtering result, and it can be seen that the amplitude and phase are also reasonably recovered. According to the attenuation model test with constant Q, we conclude that the LCFS method can also get relatively accurate results at times without spectrum information, and more reasonable results can be obtained using a reasonable time-frequency transform.

The traditional methods for estimating Q-factors in the time-frequency domain need to extract spectral information from the target layers top and bottom interfaces. In contrast, the method proposed in this paper only needs to extract information from a reference interface. We compare the proposed method with the traditional SR and CFS methods to further clarify its effect. We first extract the frequency spectrum of the corresponding time at the maximum amplitude of the six interfaces (extracted in Figure 5b) and estimate the Q-factors using the SR and CFS methods and interpolate them. The results are shown in the red lines in Figures 7a and 7b, respectively (the black dotted line represents the theoretical value). Then, we change the picking time and estimate the Q-factors using the SR (Figure 7c) and CFS (Figure 7d) methods. In Figure 7, we can see that the conventional Q-estimation methods need to pick the reflection interface accurately, but it is difficult to achieve in the actual seismic data processing.

We estimated Q-factors in noise-added attenuated signals to confirm the stability and effectiveness of the LCFS method. Random noise of different levels is added to the attenuated signal, as shown in Figures 8a, 8d, and 8g, respectively. The time-frequency distributions (Figures 8b, 8e, and 8h) of the noise-added signals are calculated using the LTFT method, wherein the black lines represent the local centroid frequencies. The red lines in Figures 8c, 8f, and 8i represent the time-varying Q-factors estimated using the proposed method. From the processing results of the noise-added signals, the LCFS method can also obtain relatively stable Q-factors in the presence of noise.

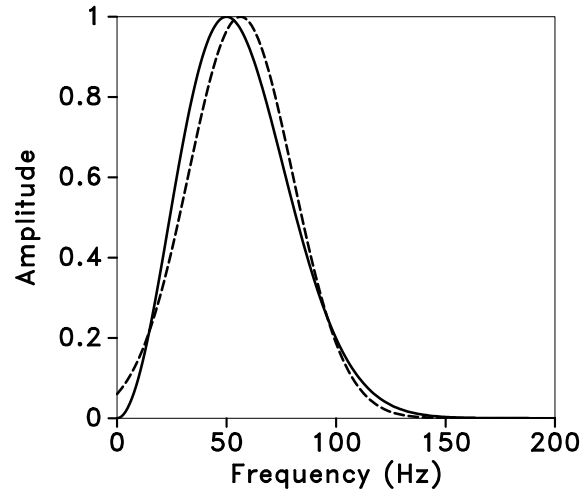


Figure 4: Amplitude spectrum of the Ricker wavelet (solid line) and the Gaussian spectrum (dotted line).

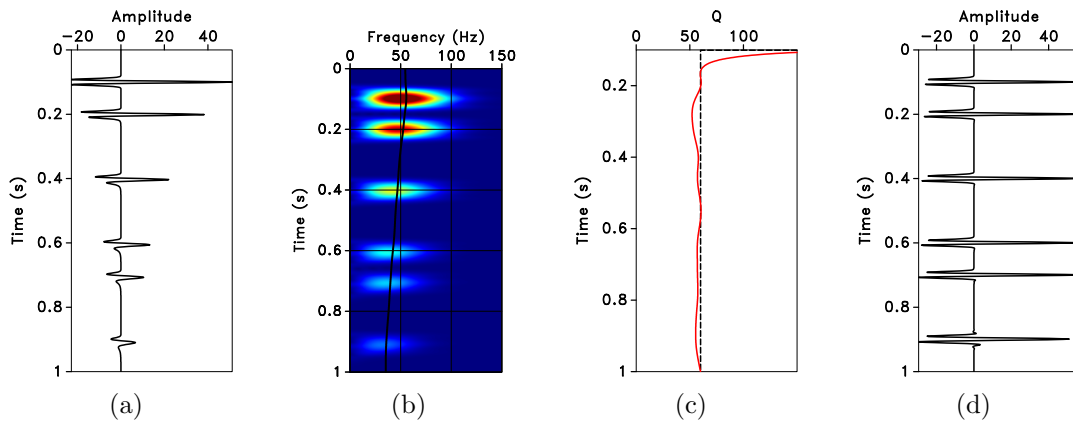


Figure 5: Attenuated model and estimated results. The attenuated model with constant  $Q$  (a), the time-frequency spectrum of the LTFT (the black line represents the local centroid frequency) (b),  $Q$  estimated using the LCFS method (red line) and theoretical  $Q$  (black dotted line) (c), inverse  $Q$  filtering result (d).

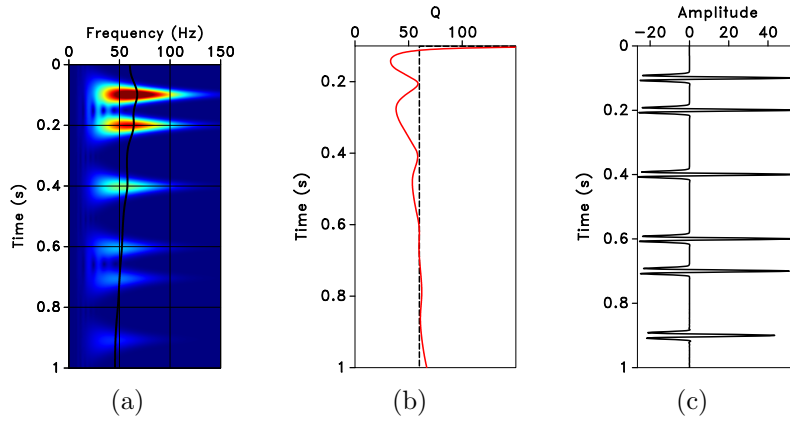


Figure 6: Results estimated using the S-transform. The time-frequency spectrum of the S-transform (the black line represents the local centroid frequency) (a),  $Q$  estimated using the LCFS method (red line) and theoretical  $Q$  (black dotted line) (b), inverse  $Q$  filtering result (c).

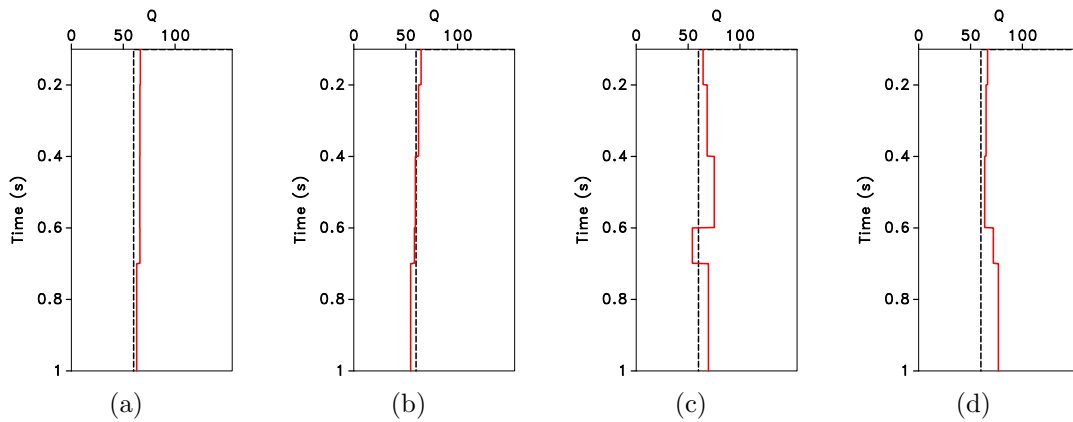


Figure 7: Estimated  $Q$ -factors using different methods (red line: estimated  $Q$ -factors; black dotted lines: theoretical  $Q$ -factors). The SR method (maximum amplitude) (a), CFS method (maximum amplitude) (b), SR method (nonmaximum amplitude) (c), CFS method (nonmaximum amplitude) (d).

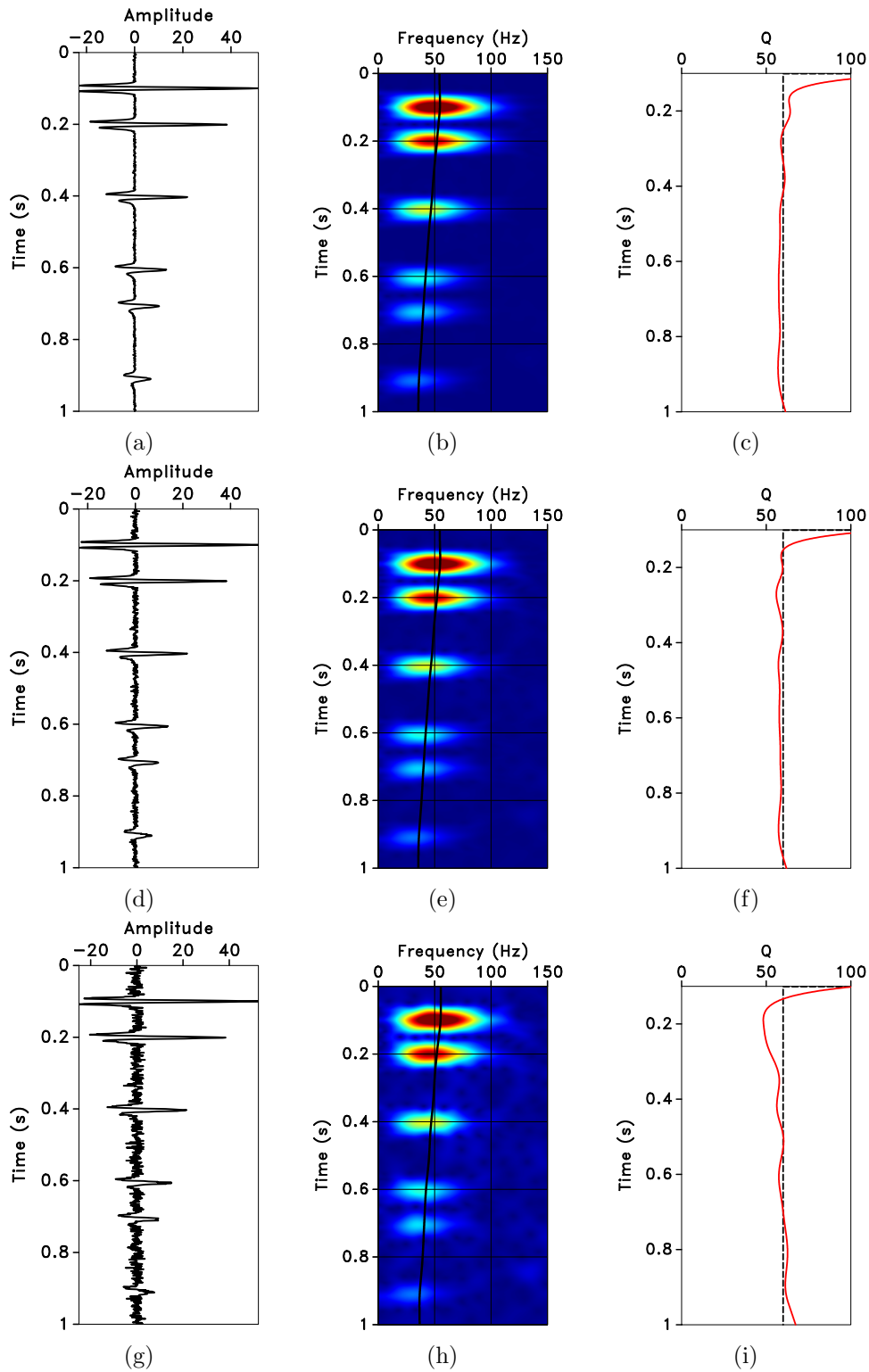


Figure 8: Noise-added attenuated signal and estimated results. The noise-added attenuated signal, and noise intensity increases gradually (a), (d), (g), the time-frequency spectrum of the LTFT (black lines represent the local centroid frequencies) (b), (e), (h),  $Q$  estimated using the LCFS method (red line) and theoretical  $Q$  (black dotted line) (c), (f), (i).

## Attenuated model with variable Q-factors

We synthesized an attenuated model with variable Q-factors to further verify the applicability of the proposed method. The time sampling interval of the synthetic signal is 1 ms, and there is a reflection interface at 0.1, 0.2, 0.4, 0.6, 0.7, and 0.9 s respectively. The attenuated seismic trace is synthesized according to the variable Q model (from 0 to 0.1 s, Q is infinite; from 0.1 to 0.2 s, Q is 50; from 0.2 to 0.4 s, Q is 80; from 0.4 to 0.6 s, Q is 30; from 0.6 to 0.7 s, Q is 100; from 0.7 to 0.9 s, Q is 120), as shown in Figure 9a. The timefrequency spectrum (Figure 9b) and the local centroid frequency (black line in Figure 9b) are obtained using the LTFT method. The results of the time-varying Q values (as shown in Figure 9c, the green dotted line represents the theoretical equivalent Q, the red line represents the estimated time-varying equivalent Q, the black dotted line represents the theoretical interval Q, and the purple line represents the time-varying interval Q calculated using the adjacent time information) using the LCFS method show that the error of the time-varying equivalent Q-factors estimated using this method is small and close to the trend of the theoretical equivalent Q-factors. However, the error of the time-varying interval Q-factors is large except for the third layer, which is mainly caused by the instability of the traditional Q-factor inversion method. Figure 9d shows the inverse Q filtering result calculated according to the time-varying equivalent Q, and the amplitude and phase of the wavelet are well compensated.

As with the attenuated model with constant Q, the proposed method is compared with the SR and CFS methods. We extract the spectrum information at the moment of the maximum amplitude in Figure 9b and estimate the interval Q-factors using the SR and CFS methods, as shown by the purple line in Figures 10a and 10b. Then, we change the picking time of the spectrum information. The Q-factors estimated using the SR and CFS methods are shown as purple lines in Figures 10c and 10d, respectively. It can be seen that the accuracy of the interface picking has a great influence on the estimated result of Q. The equivalent Q-factors are calculated according to the estimated interval Q-factors (the red lines in Figures 10a, 10b, 10c, and 10d). Compared with the time-varying equivalent Q-factors estimated using the LCFS method, the error of the traditional method is larger. The CFS method can also calculate continuous Q-factors without the extraction of interfaces. Figure 10e is the time-frequency spectrum of the attenuated signal (Figure 9a), and the black line represents the instantaneous centroid frequency calculated using equation 3. We select the instantaneous centroid frequency at 0.1 s as the reference value and calculate the time-varying equivalent Q-factors (red line in Figure 10f) and the time-varying interval Q-factors (purple line in Figure 10f) based on the instantaneous centroid frequency. Compared with the LCFS method, this method can only obtain relatively accurate equivalent Q-factors when the spectrum is available. Moreover, the estimated time-varying interval Q-factors are extremely unstable.

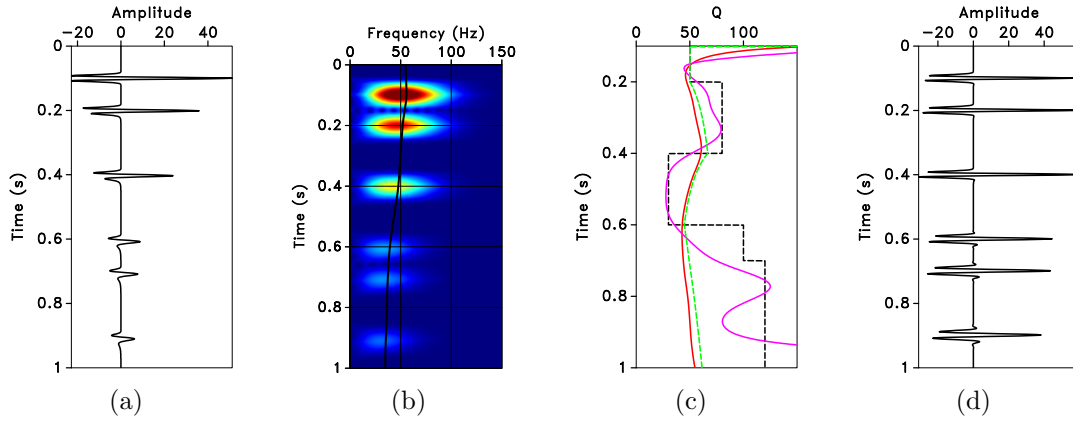


Figure 9: Attenuated model and estimated results. The attenuated model with variable  $Q$  (a), the time-frequency spectrum of the LTFT (the black line represents the local centroid frequency) (b),  $Q$  estimated using the LCFS method (the green dotted line represents the theoretical equivalent  $Q$ , the red line represents the estimated time-varying equivalent  $Q$ , black dotted line represents the theoretical interval  $Q$ , and the purple line represents the estimated time-varying interval  $Q$  calculated) (c), inverse  $Q$  filtering result (d).

## Field data

For the field data test, we select a 2D poststack seismic section (Figure 11a) that contains 201 seismic traces, each of which has a time length of 3 s and a time sampling interval of 4 ms. For comparison, we perform the automatic gain control (AGC) processing on the original seismic section. The processed result after AGC is shown in Figure 11b. AGC can enhance the weak amplitude of the deep part, but it cannot improve the resolution of the deep data. Next, we estimate the time-varying equivalent  $Q$ -factors for each seismic trace using the LCFS method and select the time corresponding to the maximum value of the local centroid frequency as the reference time. The estimated equivalent  $Q$  profile is smoothed to some extent in the spatial direction to preserve the lateral continuity. Figure 11c shows that the  $Q$ -factors calculated using the LCFS method have a reasonable distribution range and the characteristics of continuous variation in the direction of time and space. Figure 11d shows the seismic section after inverse  $Q$  filtering based on the time-varying equivalent  $Q$ -factors. Comparing Figures 11a, 11b, and 11d, we can see that the resolution of the deep part of the seismic data after inverse  $Q$  filtering has improved, the absorption and attenuation of the formation are reasonably compensated, and the original structural characteristics are well maintained.

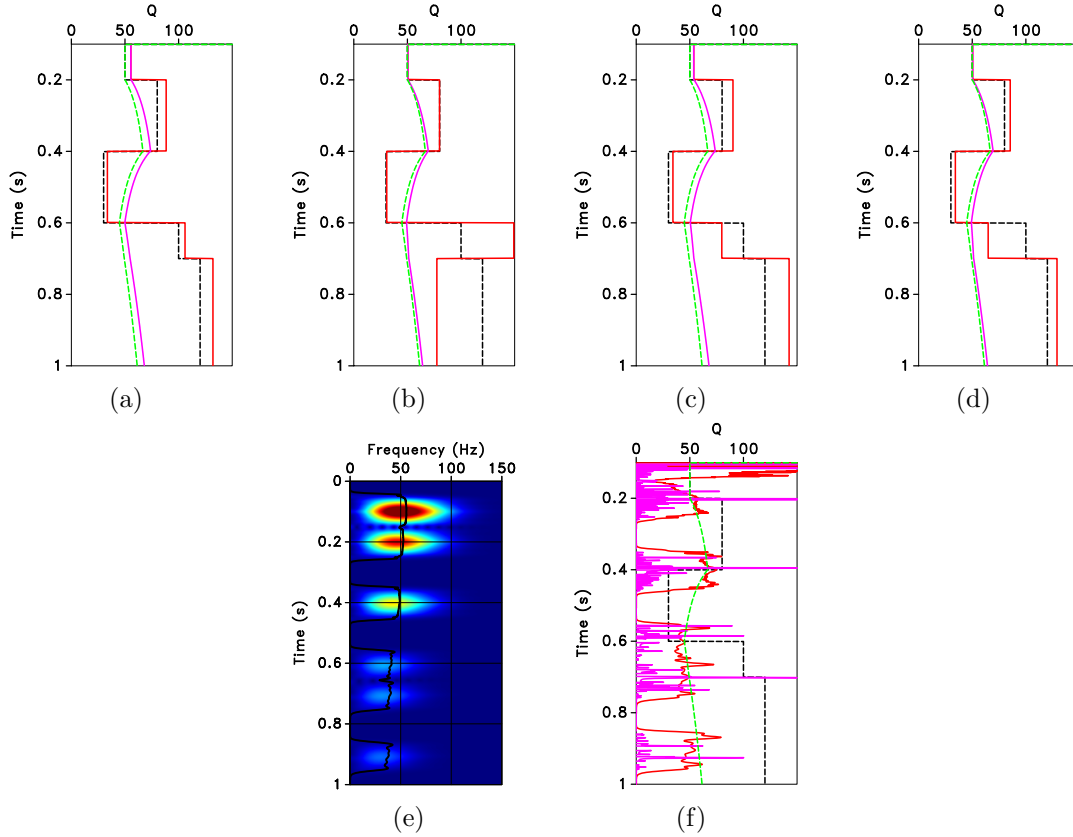


Figure 10: Q-factors estimated using different methods. The SR method (maximum amplitude) (a), CFS method (maximum amplitude) (b), SR method (nonmaximum amplitude) (c), CFS method (nonmaximum amplitude) (d), time-frequency spectrum of the LTFT (the black line represents the instantaneous centroid frequency) (e), Q estimated using the CFS method based on instantaneous centroid frequency (f).



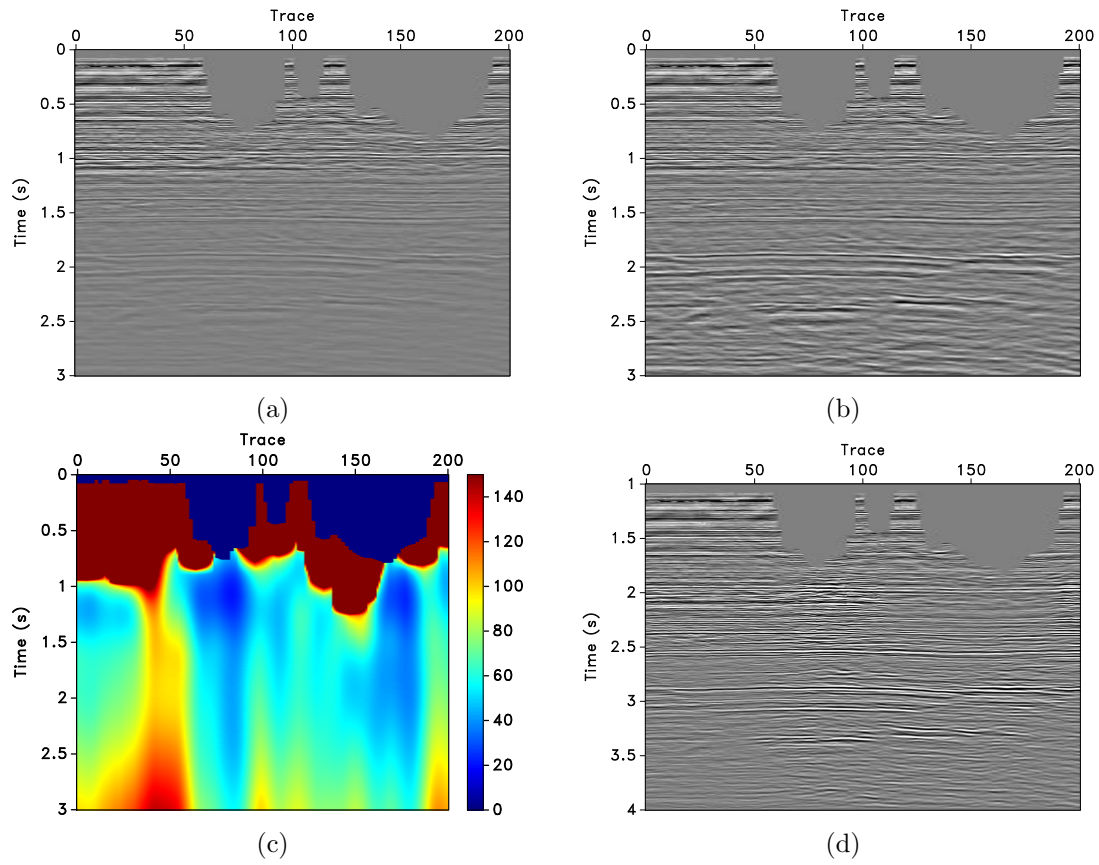


Figure 11: Processing result of field data. Poststack field data (a), the seismic data after AGC (b), the estimated time-varying equivalent Q-factors (c), the seismic data after inverse Q filtering (d).

## CONCLUSION

We define the local centroid frequency using shaping regularization in the time-frequency domain and discuss the theoretical basis of the local centroid frequency in detail in this paper. The local centroid frequency is calculated locally using information around a certain time so that a reasonable result still can be obtained at the time without information. By combining the local centroid frequency with the CFS method, we propose a continuous time-varying Q-estimation method, namely, the LCFS method. The theoretical model shows that the proposed method can obtain reasonable time-varying Q-factors even in an environment with random noise. Compared with the traditional methods, the proposed method avoids picking the target layer and can still obtain a stable estimation result in the absence of spectrum information. At the same time, experiments with synthetic examples and field data confirm that the inverse Q filtering method based on continuous time-varying Q-factors can better recover the phase and amplitude of the attenuation signal and improve the resolution of seismic data.

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